

Signaling Effects of Monetary Policy

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Abstract

We develop a dynamic general equilibrium model in which the policy rate signals the central bank's view about macroeconomic developments to price setters. The model is estimated with likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* as a measure of price setters' inflation expectations. This model improves upon existing perfect information models in explaining why, in the data, inflation expectations respond with delays to monetary impulses and remain disanchored for years. In the 1970s, U.S. monetary policy is found to signal persistent inflationary shocks, explaining why inflation and inflation expectations were so persistently heightened. Signaling effects of monetary policy also explain why inflation expectations adjusted more sluggishly than inflation after the robust monetary tightening of the 1980s.

Keywords: disanchoring of inflation expectations, heterogenous beliefs, endogenous signals, Bayesian counterfactual analysis, Bayesian VAR, dispersed information.

JEL classification: E52, C11, C52, D83.

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1 Introduction

A salient feature of economic systems is that information is dispersed across market participants and policymakers. Dispersed information implies that the publicly observed decisions implemented by these policymakers convey information to market participants. A canonical example is the interest rate set by a central bank. Such an information transfer may strongly influence the transmission of monetary impulses and the central bank's ability to stabilize the economy. Consider the case in which a central bank expects that an exogenous disturbance will raise inflation in the next few quarters. On the one hand, as predicted by standard macroeconomic models, tightening monetary policy has the effect of mitigating the inflationary effects of the shock. On the other hand, raising the policy rate might also cause higher inflation if this action *signals* to unaware market participants that an inflationary shock is about to hit the economy. While the first type of monetary transmission has been intensively investigated in the economic literature, the signaling effects of monetary policy have received far less attention.

This paper develops a dynamic stochastic general equilibrium (DSGE) model to study the empirical relevance of the signaling effects of monetary policy and their implications for the propagation of policy and non-policy disturbances. In the model, price-setting firms face nominal rigidities and dispersed information. Firms observe their own specific technology conveying noisy private information about aggregate technology shocks that influence the future dynamics of firms' nominal marginal costs. Furthermore, price setters observe a noisy private signal about disturbances affecting households' discount factor (henceforth, demand shocks) as well as the policy rate set by the central bank according to a Taylor-type reaction function. The policy signal provides public information about the central bank's view on current inflation and the output gap to firms. The central bank is assumed to have imperfect information and thereby can make errors in forecasting the targeted macroeconomic aggregates. We call this model the *dispersed information model* (DIM).

The DIM features two channels of monetary transmission. The first channel is based on the central bank's ability to affect the real interest rate because of both nominal rigidities and dispersed information. Changes in the real interest rate induce households to intertemporally adjust their consumption. The second channel arises because the policy rate signals non-redundant information to firms and hence directly influences their beliefs about macroeconomic developments. We label this second channel *the signaling channel* of monetary transmission. The signaling effects of monetary policy on the propagation of shocks critically depend on how price setters interpret changes in the policy rate. For instance, raising the policy rate can be interpreted by price-setting firms in two ways. First, a monetary tightening might be read as the central bank responding to an exogenous deviation from its monetary policy rule; that is, a contractionary monetary shock or an overestimation of the rate of inflation or the output gap.

Second, a higher interest rate may also be interpreted as the response of the central bank to inflationary non-policy shocks, which, in the model, are an adverse aggregate technology shock or a positive demand shock. If the first interpretation prevails among price setters, tightening (easing) monetary policy curbs (raises) firms' inflation expectations and hence inflation. If the second interpretation prevails, raising (cutting) the policy rate induces firms to expect higher (lower) inflation, and hence inflation rises (falls).

The model is estimated through likelihood methods on a U.S. data set that includes the *Survey of Professional Forecasters* (SPF) as a measure of price setters' inflation expectations. The data range includes the 1970s, which were characterized by one of the most notorious episodes of heightened inflation and inflation expectations in recent U.S. economic history. In the estimated model, firms mostly rely on private signals to learn about aggregate technology shocks. Conversely, firms receive fairly inaccurate private signals about demand shocks, forcing them to look at the policy signal to learn about these shocks. Nevertheless, the policy signal turns out to be equally informative about demand shocks and exogenous deviations from the monetary policy rule, making it hard for firms to tell these two shocks apart.

This information structure has important implications for the propagation of aggregate disturbances. The signaling effects of monetary policy bring about *deflationary* pressures in the aftermath of a *positive* demand shock. When the Federal Reserve raises the interest rate in response to a positive demand shock, firms attach some probability that both a contractionary monetary shock and a persistent overestimation of the output gap by the central bank might have occurred. These beliefs lower price setters' inflation expectations and hence inflation. Thus, the signaling channel makes demand shocks look like supply shocks that move prices and quantities in opposite directions. Unlike the technology shocks, these artificial supply shocks imply a negative comovement between the federal funds rate and the rate of inflation as well as between the federal funds rate and inflation expectations. This property of these artificial supply shocks helps the model fit the 1970s, when the nominal federal funds rate was kept relatively low and inflation expectations attained fairly high levels.

We estimate a benchmark Vector AutoRegressive (VAR) model to show that in the data inflation expectations respond to monetary impulses with delays and remain disanchored for more than five years.¹ State-of-the-art perfect information models are shown to have too weak a propagation mechanism to explain this pattern. In contrast, the estimated DIM accounts for these empirical facts remarkably well. The monetary tightening that immediately follows a contractionary monetary shock signals positive demand shocks that give rise to upward pressures on inflation expectations. These signaling effects substantially delay the short-run response

¹The VAR model is agnostic about economic theories and broadly parameterized. Therefore, this model is often used to obtain an accurate representation of the data. See Christiano, Eichenbaum and Evans (2005) and Del Negro et al. (2007) among many others.

of inflation expectations to this shock. In the longer run, the monetary tightening ends up signaling persistent underestimation of potential output by the central bank, leading to a disanchoring of inflation expectations that is remarkably similar to what is observed in the data. Furthermore, while the DIM can explain the large and persistent conditional forecast errors that are observed in the data, perfect information models cannot. In fact, a general property of perfect information models is that conditional forecast errors are always equal to zero. This property arises because the nature and the magnitude of the initial shock are perfectly known by all agents. This property is not shared by the dispersed information model in which rational agents are confused about the nature and the magnitude of realized shocks.

We find that the signaling effects of monetary policy significantly contribute to explaining why inflation and, especially, inflation expectations were so persistently heightened in the 1970s. The Federal Reserve's response to two large negative demand shocks that occurred in 1974 ended up signaling both expansionary monetary shocks and an overestimation of potential output by the central bank. These signaling effects raised inflation expectations and hence inflation throughout the second half of that decade. This econometric evaluation of the signaling effects controls for the inflationary pressures owing to the persistently large overestimation of potential output by the Federal Reserve in the 1970s, which is documented in the Federal Reserve's Greenbook.² We also show that the realization of the two large negative demand shocks in 1974 is supported by the VAR evidence once the identification of those shocks is corrected for the signaling effects of monetary policy.

This is the first paper that provides an econometric analysis on signaling effects of monetary policy based on a microfounded dynamic general equilibrium model. Using a reduced-form model, Romer and Romer (2000) and Tang (2015) find evidence of signaling effects of monetary policy in the U.S. Moreover, Nakamura and Steinsson (2015) and Campbell et al. (Forthcoming) assess the macroeconomic effects of Federal Open Market Committee's (FOMC) announcements about the future likely evolution of the federal funds rate (FOMC forward guidance). They find that FOMC forward guidance conveys FOMC's private information to market participants and this information transfer has large macroeconomic effects.³

The idea that the monetary authority sends public signals to an economy in which agents have dispersed information was pioneered by Morris and Shin (2003a, 2003b). The model studied in this paper is built on Nimark (2008). A particularly useful feature of Nimark's model is that the supply side of the model economy can be analytically worked out and characterized by an equation that nests the standard New Keynesian Phillips curve. The model studied in this paper shares this feature. Nonetheless, in Nimark (2008) the signaling channel does

²Orphanides (2001, 2002, 2003) argues that the Federal Reserve's persistent overestimation of potential output in the 1970s led to overexpansionary policies, which ultimately resulted in high inflation.

³Campbell et al. (2012) dubbed these effects as Delphic forward guidance.

not arise because assumptions about the Taylor-rule specification imply that the policy rate conveys only redundant information to price setters. We introduce a method to solve the DIM that belongs to the more general class of solution methods introduced by Nimark (2011). Our solution method improves upon the one used by Nimark (2008) in that it does not require numerically solving any nonlinear equations.

This paper is also related to a quickly growing empirical literature that uses the SPF to study the response of public expectations to monetary policy decisions. Del Negro and Eusepi (2011) perform an econometric evaluation of the extent to which the inflation expectations generated by the imperfect information model developed by Erceg and Levin (2003) are in line with the observed inflation expectations. There are two main differences between that paper and this one. First, in that paper monetary policy does not have signaling effects besides transferring information about the central bank's inflation target. Second, in our settings, price setters have heterogeneous beliefs. Coibion and Gorodnichenko (2012*b*) find that the Federal Reserve raises the policy rate more gradually if the private sector's inflation expectations are lower than the Federal Reserve's forecasts of inflation. This empirical evidence can be rationalized in a model in which monetary policy has signaling effects and the central bank acts strategically to stabilize public inflation expectations. Coibion and Gorodnichenko (2012*a*) use the SPF to document robust evidence in favor of models with informational rigidities.

This paper also belongs to a quite thin literature that carries out likelihood-based analyses on models with dispersed information. Nimark (2014*b*) estimates an island model built on Lorenzoni (2009) and augmented with man-bites-dog signals, which are signals that are more likely to be observed when unusual events occur. Maćkowiak, Moench, and Wiederholt (2009) use a dynamic factor model to estimate impulse responses of sectoral price indexes to aggregate shocks and to sector-specific shocks for a number of models, including a rational inattention model. Melosi (2014) conducts an econometric analysis of a stylized DSGE model with dispersed information à la Woodford (2002).

Bianchi and Melosi (2014*a*) develop a DSGE model that features waves of agents' pessimism about how aggressively the central bank will react to future changes in inflation to study the welfare implications of monetary policy communication. Gorodnichenko (2008) introduces a model in which firms make state-dependent decisions on both pricing and acquisition of information and shows that this model delivers a delayed response of inflation to monetary shocks. Trabandt (2007) analyzes the empirical properties of a state-of-the-art sticky-information DSGE model à la Mankiw and Reis (2002) and compares them with those of a state-of-the-art DSGE model with sticky prices à la Calvo.

The paper is organized as follows. In Section 2, we describe the dispersed information model, in which monetary policy has signaling effects, as well as a model in which firms have perfect information. In Section 3, we present the empirical analysis of the paper, including

the econometric evaluation of the signaling effects of monetary policy. In Section 4, we assess the robustness of our findings to changes in model specification. We present our conclusions in Section 5.

2 Models

Section 2.1 introduces the model with dispersed information and signaling effects of monetary policy. In Section 2.2, we present the time protocol of the model. Section 2.3 presents the problem of households. Section 2.4 presents firms' price-setting problem. In Section 2.5, the central bank's behavior and government's behavior are modeled. In Section 2.6, we introduce the information set available to firms and its rationale. Section 2.7 deals with the log-linearization and the solution of the dispersed information model. Finally, Section 2.8 presents the perfect information model, which will be used to evaluate the empirical significance of the dispersed information model.

2.1 The Dispersed Information Model (DIM)

The economy is populated by a continuum $(0, 1)$ of households, a continuum $(0, 1)$ of monopolistically competitive firms, a central bank (or monetary authority), and a government (or fiscal authority). A Calvo lottery establishes which firms are allowed to reoptimize their prices in any given period t (Calvo 1983). Households consume the goods produced by firms, demand government bonds, pay taxes to or receive transfers from the fiscal authority, and supply labor to the firms in a perfectly competitive labor market. Firms sell differentiated goods to households. The fiscal authority has to finance maturing government bonds. The fiscal authority can issue new government bonds and can either collect lump-sum taxes from households or pay transfers to households. The central bank sets the nominal interest rate at which the government's bonds pay out their return.

2.2 The Time Protocol

Any period t is divided into three stages. All actions that are taken in any given stage are simultaneous. At stage 0, the central bank sets the interest rate for the current period t using a Taylor-type reaction function and after observing an imperfect measure of current inflation and the output gap. At stage 1, firms update their information set by observing *(i)* their idiosyncratic technology, *(ii)* a private signal about the demand shocks, and *(iii)* the interest rate set by the central bank. Given these observations, firms set their prices. At stage 2, households learn about the realization of all the shocks in the economy and therefore become perfectly informed. Households then decide their consumption, C_t ; their demand for one-period

nominal government bonds, B_t ; and their labor supply, N_t . At this stage, firms hire labor and produce so as to deliver the demanded quantity at the price they have set at stage 1. The fiscal authority issues bonds and collects taxes from households or pays transfers to households. The markets for goods, labor, and bonds clear.

2.3 Households

Households have perfect information,⁴ and hence, we can use the representative household to solve their problem at stage 2 of every period t :

$$\max_{C_{t+s}, B_{t+s}, N_{t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \beta^{t+s} g_{t+s} [\ln C_{t+s} - \chi_n N_{t+s}],$$

where β is the deterministic discount factor and g_t is an exogenous variable influencing this factor. The logarithm of this exogenous variable follows an autoregressive process: $\ln g_t = \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}$ with Gaussian shocks $\varepsilon_{g,t} \sim \mathcal{N}(0, 1)$. We refer to g_t as *demand conditions* and to the innovation $\varepsilon_{g,t}$ as the *demand shock*. Disutility from labor linearly enters the period utility function. The parameter χ_n affects the marginal disutility of labor.

The flow budget constraint of the representative household in period t is given as follows:

$$P_t C_t + B_t = W_t N_t + R_{t-1} B_{t-1} + \Pi_t - T_t, \quad (1)$$

where P_t is the price level of the composite good consumed by households and W_t is the (competitive) nominal wage, R_t stands for the nominal (gross) interest rate, Π_t is the (equally shared) dividends paid out by the firms, and T_t stands for the lump-sum transfers/taxes. Composite consumption in period t is given by the Dixit-Stiglitz aggregator $C_t = \left(\int_0^1 C_{j,t}^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}$, where $C_{j,t}$ is consumption of the good produced by firm j in period t and ν is the elasticity of substitution between consumption goods.

At stage 2 of every period t , the representative household chooses its consumption of the good produced by firm j , labor supply, and bond holdings subject to the sequence of the flow budget constraints and a no-Ponzi-scheme condition. The representative household takes as given the nominal interest rate, the nominal wage rate, nominal aggregate profits, nominal lump-sum transfers/taxes, and the prices of all consumption goods. It can be shown that the demand for the good produced by firm j is:

$$C_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\nu} C_t, \quad (2)$$

⁴The main results of the paper are unlikely to change if one assumes that households have also dispersed information. See Appendix K.

where the price level of the composite good is given by $P_t = (\int (P_{j,t})^{1-\nu} di)^{\frac{1}{1-\nu}}$.

2.4 Firms' Price-Setting Problem

Firms are endowed with a linear technology $Y_{j,t} = a_{j,t}N_{j,t}$, where $Y_{j,t}$ is the output produced by the firm j at time t , $N_{j,t}$ is the amount of labor employed by firm j at time t , and $a_{j,t}$ is the firm-specific level of technology that can be decomposed into a level of aggregate technology (a_t) and a white-noise firm-specific component ($\varepsilon_{j,t}^a$). More specifically,

$$\ln a_{j,t} = \ln a_t + \tilde{\sigma}_a \varepsilon_{j,t}^a, \quad (3)$$

where $\varepsilon_{j,t}^a \stackrel{iid}{\sim} \mathcal{N}(0, 1)$ and a_t stands for the *level of aggregate technology* that evolves according to the autoregressive process $\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \varepsilon_{a,t}$ with Gaussian innovations $\varepsilon_{a,t} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. We refer to the innovation $\varepsilon_{a,t}$ as the (aggregate) *technology shock*.

Following Calvo (1983), we assume that a fraction θ of firms are not allowed to reoptimize the price of their respective goods at stage 1 of any period. Those firms that are not allowed to reoptimize are assumed to index their price to the steady-state inflation rate. Let us denote the (gross) steady-state inflation rate as π_* , the nominal marginal costs for firm j as $MC_{j,t} = W_t/a_{j,t}$, the time t value of one unit of the composite consumption good in period $t+s$ to the representative household as $\xi_{t|t+s}$, and the expectation operator conditional on firm j 's information set $\mathcal{I}_{j,t}$ as $\mathbb{E}_{j,t}$. The information set contains both private and public signals and will be defined in Section 2.6. At stage 1 of every period t , an arbitrary firm j that is allowed to reoptimize its price $P_{j,t}$ solves

$$\max_{P_{j,t}} \mathbb{E}_{j,t} \left[\sum_{s=0}^{\infty} (\beta\theta)^s \xi_{t|t+s} (\pi_*^s P_{j,t} - MC_{j,t+s}) Y_{j,t+s} \right],$$

subject to $Y_{j,t} = C_{j,t}$ (i.e., firms commit themselves to satisfying any demanded quantity that will arise at stage 2), to the firm j 's specific demand in equation (2), and to the linear production function. When solving the price-setting problem at stage 1, firms have to form expectations about the evolution of their nominal marginal costs, which will be realized in the next stage of the period (i.e., stage 2), using their information set $\mathcal{I}_{j,t}$. At stage 2, firms produce and deliver the quantity the representative household demands for their specific goods at the prices they set in the previous stage 1. At stage 2 we assume that firms do not receive any further information or any additional signals to what they have already observed at stage 1.

2.5 The Monetary and Fiscal Authorities

The monetary authority sets the nominal interest rate according to a Taylor-type reaction function: $R_t = (r_*\pi_*) (\tilde{\pi}_t/\pi_*)^{\phi_\pi} (\tilde{x}_t)^{\phi_x} \xi_{m,t}$, where r_* is the steady-state real interest rate and $\tilde{\pi}_t$ is the inflation rate observed by the central bank at stage 0 of time t when it has to set the interest rate R_t . We assume that the central bank knows the current inflation rate π_t up to the realization of a random variable that follows an autoregressive process $\ln \xi_{\pi,t} = \rho_\pi \ln \xi_{\pi,t-1} + \sigma_\pi \varepsilon_{\pi,t}$ with Gaussian innovations $\varepsilon_{\pi,t} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. This exogenous process captures the central bank's nowcast errors for the inflation rate. In symbols, we write this as follows: $\tilde{\pi}_t = \pi_t \xi_{\pi,t}$. We will refer to the process $\xi_{\pi,t}$ as the *central bank's measurement error for inflation*. Analogously, \tilde{x}_t denotes the output gap when the central bank is called to set the policy rate at stage 0.⁵ We assume that the central bank knows the current output gap x_t up to the realization of a random variable that follows an autoregressive process $\ln \xi_{x,t} = \rho_x \ln \xi_{x,t-1} + \sigma_x \varepsilon_{x,t}$ with Gaussian innovations $\varepsilon_{x,t} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. This exogenous process captures the central bank's nowcast errors for the output gap. We will refer to the process $\xi_{x,t}$ as the *central bank's measurement error for the output gap*. In symbols, we write this as follows: $\tilde{x}_t = x_t \xi_{x,t}$. Furthermore, the process $\xi_{m,t}$ is an exogenous random variable that is driven by the following autoregressive process: $\ln \xi_{m,t} = \rho_m \ln \xi_{m,t-1} + \sigma_m \varepsilon_{m,t}$, with Gaussian innovations $\varepsilon_{m,t} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. We will refer to the process $\xi_{m,t}$ as the *state of monetary policy* and to the innovation $\varepsilon_{m,t}$ as the *monetary policy shock*.

It should be noted that we model policy inertia as a persistent monetary policy shock rather than adding a smoothing component. Rudebusch (2002, 2006) uses term-structure data to argue that monetary policy inertia likely reflects omitted variables in the rule and that such policy inertia can be adequately approximated by persistent shocks in the rule. Furthermore, this modeling choice serves the purpose of solving the dispersed information model fast enough to allow likelihood estimation.

The policy rule can then be rewritten as follows:

$$R_t = (r_*\pi_*) \left(\frac{\pi_t}{\pi_*} \right)^{\phi_\pi} x_t^{\phi_x} \eta_{r,t}, \quad (4)$$

where $\eta_{r,t} \equiv \xi_{m,t} \xi_{\pi,t}^{\phi_\pi} \xi_{x,t}^{\phi_x}$ captures the exogenous deviations of the interest rate from the monetary policy rule. These deviations may occur as a result of monetary policy shocks $\varepsilon_{m,t}$ or as a result of measurement errors by the central bank, $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$. We will refer to the process $\eta_{r,t}$ as the *exogenous deviation from the policy rule*.

The budget constraint of the fiscal authority in period t is represented as follows $R_{t-1}B_{t-1} -$

⁵The output gap is the difference between current output and potential output, which is defined as the level of output that would arise under perfectly flexible price ($\theta = 0$) and perfect information.

$B_t = T_t$. The fiscal authority finances maturing government bonds by either collecting lump-sum taxes or issuing new government bonds. The aggregate resource constraint implies $Y_t = C_t$.

2.6 Firms' Information Set

Firms have imperfect knowledge about the history of shocks that have hit the economy. More specifically, it is assumed that firms' information set includes the history of firm-specific technology $\ln a_{j,t}$ and the history of a private signal $g_{j,t}$ on the demand conditions g_t , which evolves according to the following process: $\ln g_{j,t} = \ln g_t + \tilde{\sigma}_g \varepsilon_{j,t}^g$, where $\varepsilon_{j,t}^g \stackrel{iid}{\sim} \mathcal{N}(0, 1)$. Moreover, firms observe the history of the nominal interest rate R_t set by the central bank, as well as the history of their own prices.⁶ To sum up, the information set $\mathcal{I}_{j,t}$ of firm j at time t is given by

$$\mathcal{I}_{j,t} \equiv \{\ln a_{j,\tau}, \ln g_{j,\tau}, R_\tau, P_{j,\tau} : \tau \leq t\}. \quad (5)$$

Firms receive the signals in $\mathcal{I}_{j,t}$ at the price-setting stage 1. We assume that firms know the structural equations of the model and its parameters. For tractability, firms use the log-linear approximation to the model structural equations around its steady-state equilibrium to solve their signal extraction problem.⁷ Finally, we assume that firms have received an infinitely long sequence of signals at any time t . This assumption substantially simplifies the task of solving the model by ensuring that the Kalman gain matrix is time invariant and the same across firms.

We follow the imperfect-common-knowledge literature (Woodford, 2002; Adam, 2007; Nirmark, 2008) in modeling the highly complex process of acquiring the relevant information by price setters, which includes information about endogenous variables other than the policy rate, such as the quantities sold by firm j ($C_{j,t}$), NIPA statistics with some lags, etc. using a set of exogenous private signals ($\hat{a}_{j,t}$ and $\hat{g}_{j,t}$).⁸ These exogenous signals are assumed to be idiosyncratic to capture the idea that price setters may pay attention to different indicators. We partially depart from this literature as we do not allow firms to observe private signals on all five exogenous state variables, which also include the three subcomponents ($\xi_{m,t}$, $\xi_{\pi,t}$, and $\xi_{x,t}$) of the overall state of monetary policy $\eta_{r,t}$. Allowing firms to observe specific exogenous

⁶Observing the history of their own price $\{P_{j,\tau} : \tau \leq t\}$ conveys only redundant information to firms because their price is either adjusted to the steady-state inflation rate, which is known by firms, or a function of the history of the signals that have been already observed in the past. Thus, this signal does not play any role in the formation of firms' expectations and will be called the redundant signal. Henceforth, when we refer to signals, we mean only the non-redundant signals (namely, $\ln A_{j,t}$, $\ln g_{j,t}$, and $\ln R_t$).

⁷The log-linearized equations will be shown in the next section.

⁸In this respect, an important advantage of the rational inattention literature (e.g., Sims 2003, 2006, 2010; Mackowiak and Wiederholt 2009) is to go beyond this reduced-form approach by allowing agents to optimally choose their signal structure under an information-processing constraint that limits the overall amount of information the signals can convey. Nonetheless, estimating a rational inattention model is not feasible at this stage because solving the problem of how firms allocate their attention optimally would increase even more the already heavy computational burden that characterizes the solution of the DIM.

signals on the central bank’s measurement errors (i.e., $\xi_{\pi,t}$ and $\xi_{x,t}$) would imply allowing firms to have an information advantage about the central bank’s measurement errors over the central bank itself. This assumption is clearly controversial. In the model price setters know the law of motion of the central bank’s measurement errors but they have to learn the magnitude thereof in every period. A less controversial assumption is to endow the firms also with a private signal about the exogenous deviations from the policy rule $\eta_{r,t}$. However, the estimated value for the noise variance of this additional private signal turns out to be so large to become non-identifiable. The presence of a non-identifiable parameter also affects the convergence of the estimation procedure for the other parameters. Thus, we did not include this additional signal to firms’ information set. It should also be emphasized that our information structure follows Woodford (2002) in assuming that firms observe a truth-plus-white-noise type of signals with serially uncorrelated noise shocks. This signal structure is arguably quite restrictive parametrically. However, these restrictions are crucial to avoid weak identification of the model parameters.

A novel ingredient of the model is to allow firms to perfectly observe the interest rate set by the central bank R_t . This assumption is based on the fact that the monetary policy rate is measured very accurately in real time and is subject neither to revisions nor to delays in reporting. These features do not extend to other aggregate endogenous variables, such as inflation or output (e.g., GDP). Moreover, Andrade et al. (2014) document that the *Blue Chip Financial Forecasts* show very small disagreement on the next quarter’s federal funds rate compared with other leading macroeconomic aggregates, such as inflation and GDP.

In Section 4 we will show that the maintained information structure in (5) delivers quite plausible dynamics for inflation nowcast errors in the estimated DIM. Furthermore, we will also show that assuming that firms observe other endogenous variables, such as the quantity firms have sold, turns out to substantially deteriorate the fit of the dispersed information model.

2.7 Log-linearization and Model Solution

We solve the firms’ and households’ problems, described in Sections 2.3 and 2.4, and obtain the consumption Euler equation and the price-setting equation. We denote the log-deviation of an arbitrary (stationary) variable x_t from its steady-state value as \hat{x}_t . As in Nimark (2008), we obtain the imperfect-common-knowledge Phillips curve that is given as follows:⁹

$$\hat{\pi}_t = (1 - \theta) (1 - \beta\theta) \sum_{k=1}^{\infty} (1 - \theta)^{k-1} \widehat{mc}_{t|t}^{(k)} + \beta\theta \sum_{k=1}^{\infty} (1 - \theta)^{k-1} \widehat{\pi}_{t+1|t}^{(k)}. \quad (6)$$

⁹See Appendix A for a detailed derivation.

In this equation, $\widehat{\pi}_{t+1|t}^{(k)}$ denotes the average k -th order expectations about the next period's inflation rate, $\widehat{\pi}_{t+1}$, that is, $\widehat{\pi}_{t+1|t}^{(k)} \equiv \underbrace{\int \mathbb{E}_{j,t} \dots \int \mathbb{E}_{j,t} \widehat{\pi}_{t+1} dj \dots dj}_k$, for any integer $k > 1$. Moreover,

$\widehat{m}c_{t|t}^{(k)}$ denotes the average k -th order expectations about the real aggregate marginal costs $\widehat{m}c_t \equiv \int \widehat{m}c_{j,t} dj$, which evolve according to the equation $\widehat{m}c_{t|t}^{(k)} = \widehat{y}_{t|t}^{(k)} - \widehat{a}_{t|t}^{(k-1)}$ for any integer $k > 1$. The imperfect-common-knowledge Phillips curve makes it explicit that price setters *forecast the forecasts* of other price setters (Townsend 1983a, 1983b). The Calvo parameter θ determines the structure of weights for the higher-order expectations in the averages $\sum_{k=1}^{\infty} (1-\theta)^{k-1} \widehat{m}c_{t|t}^{(k)}$ and $\sum_{k=1}^{\infty} (1-\theta)^{k-1} \widehat{\pi}_{t+1|t}^{(k)}$. The smaller the Calvo parameter, the more the model dynamics are affected by the the average expectations of relatively higher orders.

The log-linearized Euler equation is standard and reads as follows:

$$\widehat{g}_t - \widehat{y}_t = \mathbb{E}_t \widehat{g}_{t+1} - \mathbb{E}_t \widehat{y}_{t+1} - \mathbb{E}_t \widehat{\pi}_{t+1} + \widehat{R}_t, \quad (7)$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation operator conditional on the complete information set. The log-linearized central bank's reaction function (4) can be written as follows:

$$\widehat{R}_t = \phi_{\pi} \widehat{\pi}_t + \phi_x (\widehat{y}_t - \widehat{a}_t) + \widehat{\eta}_{r,t}, \quad (8)$$

where $\widehat{y}_t - \widehat{a}_t$ is the log output gap.

The demand conditions evolve according to $\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \sigma_g \varepsilon_{g,t}$. The process for aggregate technology becomes $\widehat{a}_t = \rho_a \widehat{a}_{t-1} + \sigma_a \varepsilon_{a,t}$. The exogenous process that leads the central bank to deviate from the monetary rule is defined as $\widehat{\eta}_{r,t} = \widehat{\xi}_{m,t} + \phi_{\pi} \widehat{\xi}_{\pi,t} + \phi_x \widehat{\xi}_{x,t}$. The subcomponents of $\widehat{\eta}_{r,t}$ evolve as follows: $\widehat{\xi}_{i,t} = \rho_i \widehat{\xi}_{i,t-1} + \sigma_i \varepsilon_{i,t}$ with $i \in \{m, \pi, x\}$. We log-linearize the signal equation concerning the level of aggregate technology (3) and obtain $\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a$. The signal equation concerning the demand conditions is written $\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g \varepsilon_{j,t}^g$. The policy signal \widehat{R}_t evolves according to equation (8).

A detailed description of how we solve the model is provided in Appendix B. The proposed solution algorithm improves upon the one used in Nimark (2008) as our approach does not require solving a system of nonlinear equations.¹⁰ When the model is solved, the law of motion

¹⁰Nimark (2014a) introduces a method to improve the efficiency of these types of solution methods for dispersed information models in which agents (e.g., firms) use lagged endogenous variables to form their beliefs. An alternative solution algorithm based on rewriting the equilibrium dynamics partly as a moving-average process and setting the lag with which the state is revealed to be a very large number is analyzed by Hellwig (2002) and Hellwig and Vankateswaran (2009). Rondina and Walker (2012) study a new class of rational expectations equilibria in dynamic economies with dispersed information and signal extraction from endogenous variables.

of the endogenous variables $\mathbf{s}_t \equiv [\widehat{y}_t, \widehat{\pi}_t, \widehat{R}_t]'$ reads as follows:

$$\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}, \quad (9)$$

where $X_{t|t}^{(0:k)} \equiv [\widehat{a}_{t|t}^{(s)}, \widehat{g}_{t|t}^{(s)}, \widehat{\xi}_{m,t|t}^{(s)}, \widehat{\xi}_{\pi,t|t}^{(s)}, \widehat{\xi}_{x,t|t}^{(s)} : 0 \leq s \leq k]'$ is the vector of the average expectations of any order from zero through the truncation $k > 0$ about the exogenous state variables $X_t = (\widehat{a}_t, \widehat{g}_t, \widehat{\xi}_{m,t}, \widehat{\xi}_{\pi,t}, \widehat{\xi}_{x,t})$. The average s -th order expectations about the level of aggregate technology, $\widehat{a}_{t|t}^{(s)}$, are defined as the integral of firms' expectations about the average $(s-1)$ -th order expectations across firms. In symbols, this is given as follows: $\widehat{a}_{t|t}^{(s)} = \int \mathbb{E}_{j,t}(\widehat{a}_{t|t}^{(s-1)}) dj$, for $1 \leq s \leq k$, where conventionally $\widehat{a}_{t|t}^{(0)} = \widehat{a}_t$. The average expectations about the demand conditions (\widehat{g}_t), the state of monetary policy ($\widehat{\xi}_{m,t}$), and the central bank's measurement errors for inflation ($\widehat{\xi}_{\pi,t}$) and for the output gap ($\widehat{\xi}_{x,t}$) are analogously defined. Note that in order to keep the dimensionality of the state vector finite, we truncate the infinite hierarchy of average higher-order expectations, considering only orders smaller than or equal to twenty. The vector of average expectations about the exogenous state variables $X_{t|t}^{(0:k)}$ is assumed to follow a VAR model of order one:¹¹

$$X_{t|t}^{(0:k)} = \mathbf{M}X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t. \quad (10)$$

We solve the model by guessing and verify the dynamics of higher-order beliefs (i.e., the matrices \mathbf{M} and \mathbf{N}). However, solving for higher-order expectations is nothing else than a particular solution method in the context of this paper. There exist other approaches that rely on the fact that average first-order expectations about the endogenous variables can be computed given the guessed laws of motion of the endogenous variables by using the assumption of rational expectations. In this case, the problem of solving the model boils down finding a fixed point over the parameters that characterizes the laws of motion for the endogenous variables of interest. See Maćkowiak and Wiederholt (2009) for an example of how this type of solution method works. When applied to our model, that approach turns out to be harder to combine with the estimation procedure (i.e., the Metropolis-Hastings posterior simulator), which requires a high degree of automatization of the solution routine. Furthermore, studying the higher-order beliefs helps interpret some of the predictions of the model.

¹¹As is standard in the literature (e.g., Woodford 2002), we focus on equilibria where the higher-order expectations about the exogenous state variables follow a VAR model of order one. To solve the model, we also assume common knowledge of rationality. See Nimark (2008, Assumption 1, p. 373) for a formal formulation of this assumption.

2.8 The Perfect Information Model (PIM)

If firms were perfectly informed, higher-order uncertainty would fade away (i.e., $X_{t|t}^{(k)} = X_t$ for any integer $k > 0$) and the linearized model would boil down to a prototypical three-equation New Keynesian DSGE model (e.g., Rotemberg and Woodford 1997; Lubik and Schorfheide 2004; Rabanal and Rubio-Ramírez 2005). Unlike in the dispersed information model, we add an exogenous process affecting the price markup so as to avoid stochastic singularity of this model, which would preclude estimation. The exogenous markup evolves according to the autoregressive process $\widehat{\xi}_{p,t} = \rho_p \widehat{\xi}_{p,t-1} + \sigma_p \varepsilon_{p,t}$ with Gaussian innovations $\varepsilon_{p,t} \stackrel{iid}{\sim} \mathcal{N}(0, 1)$.¹² The new Keynesian Phillips curve is given as follows: $\widehat{\pi}_t = \kappa_{pc} \widehat{m}c_t + \beta \mathbb{E}_t \widehat{\pi}_{t+1} + \widehat{\xi}_{p,t}$, where $\kappa_{pc} \equiv (1 - \theta)(1 - \theta\beta) / \theta$ with the real marginal costs given by $\widehat{m}c_t = \widehat{y}_t - \widehat{a}_t$. To improve the empirical performance of this alternative model, we assume that households' utility is affected by consumption habits. The Euler equation for consumption is as follows: $\widehat{c}_t = h(1 + h)^{-1} \widehat{c}_{t-1} + (1 + h)^{-1} E_t \widehat{c}_{t+1} - (1 - h)(1 + h)^{-1} \widehat{R}_t + (1 - h)(1 + h)^{-1} E_t \widehat{\pi}_{t+1} + (1 - h)(1 + h)^{-1} (1 - \rho_g) \widehat{g}_t$. The Taylor rule is the same as in the dispersed information model. We call this prototypical New Keynesian DSGE model the perfect information model (PIM).

3 Empirical Analysis

This section contains the econometric analysis of the model and the signaling channel of monetary policy. In Section 3.1, we present the data set. In Section 3.2, we discuss the prior and posterior distribution for the model parameters. In Section 3.3, we evaluate the ability of the DIM to fit the data relative to that of the PIM. In Section 3.4, we assess the relative ability of the DIM to replicate a few stylized empirical facts about the propagation of monetary impulses. In Section 3.5, we study the propagation of the structural disturbances in the estimated DIM. In Section 3.6, we run a Bayesian counterfactual experiment to assess the empirical relevance of the signaling effects of monetary policy.

3.1 The Data

The model is estimated using a data set that comprises the following seven observable variables for the U.S. economy: the Hodrick-Prescott (HP) filtered output gap,¹³ the inflation rate (GDP deflator), the federal funds rate, one- and four-quarter-ahead inflation expectations from the

¹²In our estimation we use data on both the output gap and inflation. In the absence of price markup shocks, it is well-known that the three-equation perfect information model features almost perfect correlation between the output gap and inflation, causing the model to be stochastically singular. Adding a markup shock loosens this tight relation between the output gap and inflation, allowing us to estimate the perfect information model.

¹³The results are robust if one computes the potential output using a quadratic trend or the output gap computed by the Congressional Budget Office.

Survey of Professional Forecasters (SPF), the real-time output gap, and real-time inflation from the Federal Reserve’s Greenbook. The data are quarterly and run from 1970:Q3 through 2007:Q4. The measurement equations and more details on how the observables are constructed are available in Appendix F. We use the SPF data to inform the average first-order expectations about inflation; that is, $\hat{\pi}_{t+1|t}^{(1)}$ and $\hat{\pi}_{t+4|t}^{(1)}$. To avoid stochastic singularity, we assume that the two series for inflation expectations are observed with i.i.d. Gaussian measurement errors. We use the real-time data on the output gap and inflation to inform the central bank’s perceived output gap $\hat{y}_t - \hat{a}_t + \hat{\xi}_{x,t}$ and its perceived inflation rate $\hat{\pi}_t + \hat{\xi}_{\pi,t}$, respectively. These series were constructed by Orphanides (2004) until 1995:Q4. We have completed the data set using the tables kept by the Federal Reserve Bank of Philadelphia after harmonizing it.¹⁴

3.2 Bayesian Estimation

As is standard, we fix the value for β so that the steady-state real interest rate is broadly consistent with its sample average. The prior and posterior statistics for the model parameters are reported in Table 1. The prior distribution for the Calvo parameter θ is centered at zero and its variance is sufficiently large to make our a-priori view about this parameter value fairly agnostic. As it will become clear, the degree of persistence of signaling effects ultimately hinges on the persistence of the shocks that the monetary authority signals to firms by changing the policy rate. Therefore, the priors for the autoregressive parameters $\rho_a, \rho_g, \rho_m, \rho_\pi,$ and ρ_x are set to be broad enough to accommodate a wide range of persistence degrees for the five exogenous processes. The values of the volatilities for the structural innovations ($\sigma_a, \sigma_g, \sigma_m, \sigma_\pi,$ and σ_x) are also crucial as they affect firms’ signaling extraction problem. Hence, we select quite broad priors for those volatilities. The noise variances of the exogenous private signals regarding aggregate technology and demand conditions ($\tilde{\sigma}_a$ and $\tilde{\sigma}_g$) are crucial for the macroeconomic implications of the signaling channel as they affect the accuracy of private information and, hence, to what extent firms rely on the policy signal to learn about these non-policy shocks. To avoid determining *a-priori* how strongly the signaling channel influences firms’ beliefs, we set a loose prior over these parameters. Finally, the prior means for the measurement errors associated with inflation expectations is set so as to match the variance of inflation expectations reported in the *Livingston Survey* following the practice of Del Negro and Schorfheide (2008).

We combine the prior distribution for the parameters of the two models (i.e., the DIM and the PIM) with their likelihood function and conduct Bayesian inference. As explained in

¹⁴The Federal Reserve Bank of Philadelphia computes the real-time output gap as percent deviations of output Y_t from its potential Y_t^* (i.e., $100(Y_t - Y_t^*)/Y_t^*$). Therefore, these data must be adjusted so as to make them consistent with the data set constructed by Orphanides (2004) for the earlier quarters and with the model’s concept of the output gap (i.e., $100(\ln Y_t - \ln Y_t^*)$). Analogous transformation is made for the real-time inflation rate.

Name	DIM - Posterior			PIM - Posterior			Type	Prior	
	Mean	5%	95%	Mean	5%	95%		Mean	Std.
θ	0.3608	0.3137	0.4112	0.4622	0.4061	0.5135	\mathcal{B}	0.50	0.30
ϕ_π	1.6782	1.4454	2.1392	1.5560	1.3770	1.7519	\mathcal{G}	1.50	0.40
ϕ_x	0.6731	0.4898	0.7917	0.0092	0.0004	0.0195	\mathcal{G}	0.50	0.40
h	—	—	—	0.1629	0.0795	0.2414	\mathcal{B}	0.50	0.20
ρ_a	0.9764	0.9635	0.9897	0.6043	0.5112	0.6886	\mathcal{B}	0.50	0.20
ρ_g	0.9038	0.8663	0.9207	0.9721	0.9576	0.9873	\mathcal{B}	0.50	0.20
ρ_m	0.9468	0.8807	0.9748	0.3483	0.2834	0.4112	\mathcal{B}	0.50	0.20
ρ_π	0.3411	0.2472	0.4577	0.2357	0.1409	0.3276	\mathcal{B}	0.50	0.20
ρ_x	0.9541	0.9311	0.9812	0.9641	0.9379	0.9897	\mathcal{B}	0.50	0.20
ρ_p	—	—	—	0.9952	0.9897	0.9995	\mathcal{B}	0.50	0.20
$100\sigma_a$	1.4208	0.9764	2.0395	0.5197	0.4437	0.5980	\mathcal{IG}	0.80	1.50
$100\tilde{\sigma}_a$	2.6068	1.5364	3.3252	—	—	—	\mathcal{IG}	0.80	1.50
$100\sigma_g$	3.6786	2.8764	4.0607	1.4489	0.6312	2.5713	\mathcal{IG}	0.80	1.50
$100\tilde{\sigma}_g$	34.884	34.240	35.522	—	—	—	\mathcal{IG}	0.80	1.50
$100\sigma_m$	0.8474	0.6866	0.9842	0.5266	0.4545	0.6030	\mathcal{IG}	0.80	1.50
$100\sigma_\pi$	0.2686	0.2415	0.3043	0.2567	0.2284	0.2849	\mathcal{IG}	0.80	1.50
$100\sigma_x$	1.0448	0.9278	1.1762	1.0442	0.9260	1.1633	\mathcal{IG}	0.80	1.50
$100\sigma_p$	—	—	—	0.5796	0.4226	0.7605	\mathcal{IG}	0.80	1.50
$100\sigma_{\mu_1}$	0.1226	0.1088	0.1388	0.1109	0.0959	0.1262	\mathcal{IG}	0.10	0.08
$100\sigma_{\mu_2}$	0.1087	0.0963	0.1215	0.0658	0.0540	0.0788	\mathcal{IG}	0.10	0.08
$100\ln \pi_*$	0.6532	0.5661	0.7482	0.8651	0.7562	0.9591	\mathcal{N}	0.65	0.10

Table 1: Prior and posterior statistics for the parameters of the dispersed information model (DIM) and the perfect information model (PIM)

Fernández-Villaverde and Rubio-Ramírez (2004) and An and Schorfheide (2007), a closed-form expression for the posterior distribution is not available, but we can approximate the moments of the posterior distribution via the Metropolis-Hastings algorithm. We obtain 250,000 posterior draws for the dispersed information model and 1,000,000 draws for the perfect information model. As far as the DIM is concerned, the posterior mean for the Calvo parameter θ implies very flexible price contracts, whose implied duration is roughly half a year. Such a small value for the Calvo parameter implies that the average expectations of relatively higher order play an important role for the macroeconomic dynamics, as discussed in Section 2.7.

The posterior mean for the inflation coefficient of the Taylor rule (ϕ_π) is higher than its prior mean and quite similar across models. The output gap coefficient in the Taylor rule ϕ_x is substantially larger in the DIM than in the PIM. Since the Taylor rule also plays the role of signaling equation in the DIM, a higher value for this parameter raises, all other things equal, the amount of information conveyed by the policy rate about the central bank's estimates of the output gap, which are exactly identified by the real-time data used in the estimation. On the contrary, the federal funds rate is found to respond very weakly to the output gap in the

PIM. The other Taylor rule’s parameters are very similar across the two models with the only exception of the persistence of monetary shocks ρ_m , which is substantially larger in the DIM. Note that highly persistent monetary shocks have the effect of increasing the persistence of the signaling effects of monetary policy on the macroeconomy insofar as changes in the policy rate signal this type of shock. It should also be noted that the autoregressive parameter for the price markup ρ_p in the PIM is estimated to be very close to unity, highlighting serious shortcomings of the PIM when it comes to endogenously accounting for the persistent dynamics of inflation in the data. This is a point to which we will return in the next section.¹⁵

The posterior mean for the variance of the firm-specific technology shock $\tilde{\sigma}_a$ implies that the posterior mean of the signal-to-noise ratio $\sigma_a/\tilde{\sigma}_a$ is 0.54. The posterior mean for the signal-to-noise ratio $\sigma_g/\tilde{\sigma}_g$ is extremely small, suggesting that firms’ private information is less accurate about demand shocks than about aggregate technology shocks. The posterior distribution implies that the following properties characterize the estimated information structure. First, firms learn mostly about aggregate technology from their private signal: the posterior median for the ratio of private information to public information about the aggregate technology is 88 percent.¹⁶ Second, firms largely rely on the policy signal \hat{R}_t to learn about the demand conditions \hat{g}_t , since the private signal conveys only 21 percent of the overall information firms gather about this exogenous state variable. Third, the policy signal conveys roughly the same amount of information about the demand shocks ($\hat{\varepsilon}_{g,t}$) and the exogenous deviations from the policy rule ($\hat{\varepsilon}_{m,t}, \hat{\varepsilon}_{\pi,t}, \hat{\varepsilon}_{x,t}$). The second property of the estimated information structure implies that firms rely mostly on the public signal to learn about the demand shocks and the exogenous deviations from the Taylor rule. However, the third property implies that firms find it hard to tell whether observed changes in the policy rate are due to exogenous deviations from the policy rule or are instead due to the central bank’s response to demand shocks. This feature is crucial to understanding most of the analysis that follows.

¹⁵Interestingly, the estimated steady-state inflation rate π_* is 20 basis points lower in the DIM than in the PIM. In both models, steady-state inflation affects only the intercept of the measurement equations for the following observables: inflation, the federal funds rate, one- and four-quarter-ahead inflation expectations, and real-time inflation. See Appendix F. As a result, this parameter is most likely informed by the sample mean of those observables. Since we use the same data set to estimate the two models, a 20-basis-point difference in the estimated value for steady-state inflation is striking. While it is very challenging to disentangle the exact reasons behind this result, it is conceivable that the highly sluggish responses of inflation and inflation expectations to shocks to the output gap mismeasurement ($\hat{\xi}_{x,t}$) in the DIM, along with the persistent overestimation of potential output observed in the 1970s and in the 1980s, have affected the estimation of this parameter. This finding is quite interesting in light of the growing theoretical literature on trend inflation (e.g., Ascari and Sbordone 2014.)

¹⁶Appendix E shows how to use entropy-based measures to assess how much information is conveyed by signals to firms. These measures quantify information flows following a standard practice in information theory (Cover and Thomas 1991).

3.3 The Empirical Fit of the DIM

The objective of this section is to validate the DIM as a reliable modeling framework for macroeconomic analysis. To this end, we compare the goodness of fit of the DIM relative to that of the PIM, which is a prototypical New Keynesian DSGE model that has been extensively used for monetary policy analysis (e.g., Rotemberg and Woodford 1997; Clarida, Galí, and Gertler 2000; Lubik and Schorfheide 2004; Coibion and Gorodnichenko 2011).

In Bayesian econometrics, non-nested model comparison is based on computing the posterior probability of the two candidate models. The marginal likelihood is the appropriate density for updating prior probabilities over a set of models.¹⁷ Since the marginal likelihood penalizes for the number of model parameters (An and Schorfheide 2007), it can be applied to gauge the relative fit of models that feature different numbers of parameters, such as the DIM and the PIM. The DIM has a log marginal likelihood equal to -319.89, which is higher than that of the PIM (-334.95). It follows that starting with 50 percent prior probability over each of the two competing models, the posterior probability of the DIM turns out to be extremely close to one. Since the PIM has one more aggregate shock than the DIM, this result has to be interpreted as fairly strong evidence in favor of the ability of the DIM to fit the data relatively well.

3.4 VAR Evidence

To further investigate the empirical performance of the DIM relative to perfect information models, we evaluate the relative ability of this structural model to account for some key empirical facts regarding the transmission of monetary disturbances to inflation and inflation expectations. We use a VAR model to establish these facts.¹⁸ We perform Bayesian estimation of this VAR model with four lags by using the data set described in Section 3.1. The results that follow are robust to adopting the larger data set used in the influential study by Christiano, Eichenbaum and Evans (2005) along with the SPF inflation expectations described in Section 3.1. We use a unit-root prior (Sims and Zha 1998) for the parameters of this Bayesian VAR (BVAR) with a presample of six quarters. As is standard, the number of lags and the five hyperparameters pinning down the prior are chosen so as to maximize the marginal likelihood.

The upper graphs of Figure 1 show the response of inflation expectations to monetary shocks identified with sign restrictions (Uhlig 2005); that is, contractionary monetary shocks move output and inflation down and the federal funds rate up for the first five quarters. The lower graphs show the implied one-quarter-ahead (left plot) and four-quarter-ahead (right plot)

¹⁷Furthermore, Fernández-Villaverde and Rubio-Ramírez (2004) show that the marginal likelihood allows the researcher to select the best model to approximate the true probability distribution of the data-generating process under the Kullback-Leibler distance.

¹⁸The VAR model unsurprisingly attains a higher marginal likelihood than that of the two structural models, which validates the VAR model as the benchmark model from a Bayesian perspective (Schorfheide 2000).

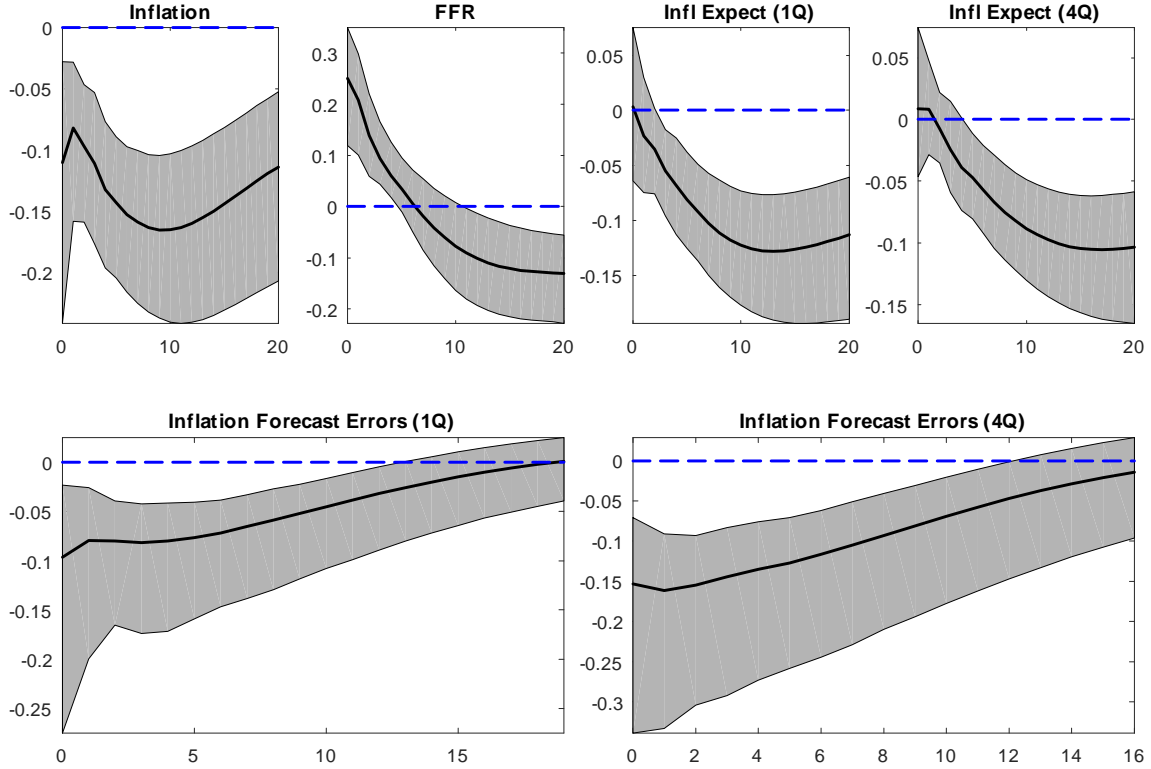


Figure 1: VAR Impulse Responses to a monetary shock and Conditional Forecast Errors. *Upper graphs:* Impulse response functions to a monetary policy shock identified with sign restrictions. *Lower graphs:* The implied one-quarter-ahead (left plot) and four-quarter-ahead (right plot) inflation forecast errors conditional on the monetary shock. Solid lines denote posterior median responses. Shaded areas denote the 70 percent posterior credible set. All numbers are annualized and in percent.

inflation forecast errors conditional on the monetary shock. The gray areas denote the 70 percent posterior credible sets and the solid line the posterior median. Three facts have to be emphasized. First, inflation forecast errors conditional on monetary shocks are fairly persistent. In the aftermath of a monetary tightening, the lower graphs of Figure 1 shows that the posterior median (the solid line) of the one-quarter-ahead and four-quarter-ahead inflation forecast errors are larger than zero for almost five years. The 70 percent posterior upper bound for these forecast errors stays in negative territory for at least three years. Second, inflation expectations barely move immediately after a monetary shock. Third, the responses of both inflation and inflation expectations to monetary shocks exhibit a great deal of persistence with half life¹⁹ exceeding 20 quarters. This last fact suggests that inflation expectations remain disanchored for a few years after a monetary contraction.

While, as we shall see, the DIM can explain large and persistent conditional forecast errors through signaling effects, perfect information models cannot. Indeed, the first fact is a conundrum for *every* perfect information model. A *general* property of perfect information models

¹⁹Half life is defined as the number quarters it takes for the largest effect of a shock to reduce to half.

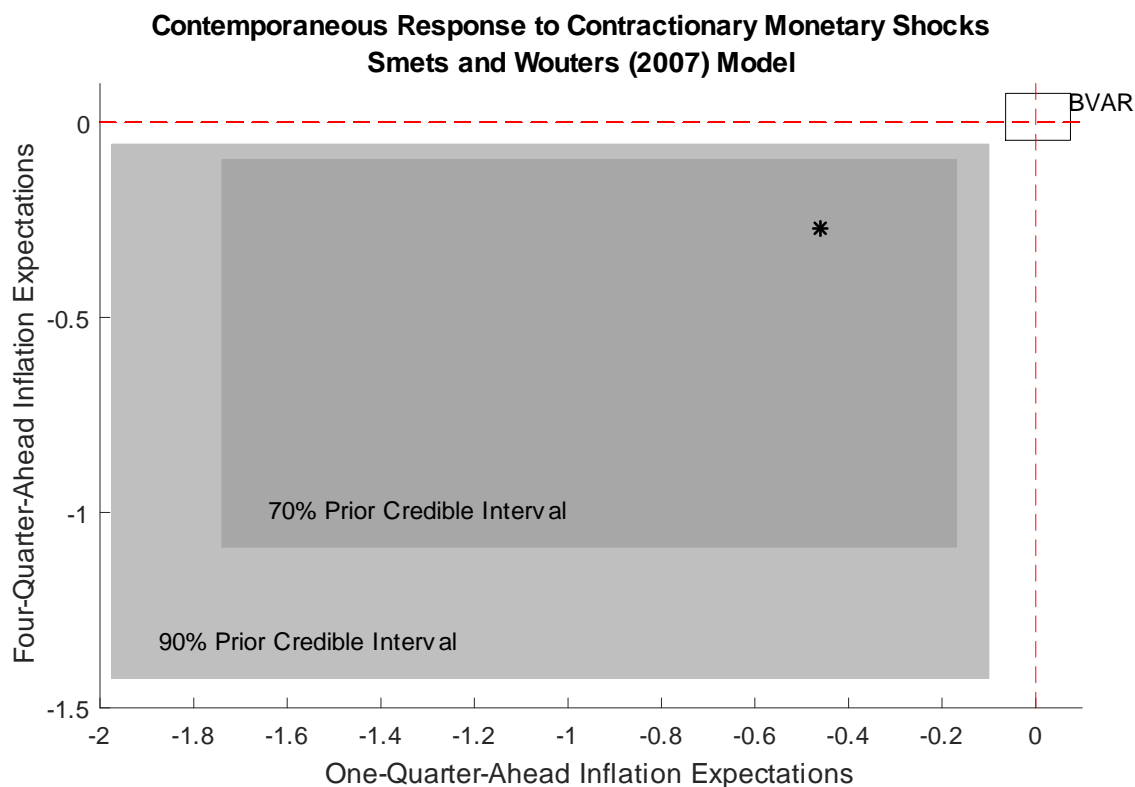


Figure 2: Smets and Wouter (2007) Model’s Prior Predictive Checks (1,000,000 draws). The dark and light gray areas denote the 70 percent- and 90 percent prior interquantile ranges for the contemporaneous response of inflation expectations at one- and four-quarter horizons to contractionary monetary shocks that raise the interest rate by 0.25 percent in the model developed by Smets and Wouters (2007). The rectangle marks the 70 percent posterior interquantile range of the VAR-implied contemporaneous responses to an equivalently scaled monetary shock. All numbers are in percent. The red dashed lines are the zero axes.

is that the response of h -period-ahead expectations turns out to be identical to the response of the actual variable h periods after the shock was realized. To put it differently, perfect information models predict conditional forecast errors to be always equal to zero. This property of perfect information models arises because the nature and the magnitude of the initial shock are perfectly known by all agents in every period. This property is not shared by the dispersed information model in which *rational* agents are confused about the nature and the magnitude of realized shocks.

State-of-the-art perfect-information models also struggle to explain the second and the third fact. Figure 2 illustrates this point. The gray areas denote the prior interquantile ranges for the contemporaneous response of inflation expectations in the Smets and Wouters (2007) model, which is a state-of-the-art New Keynesian DSGE model with perfect information.²⁰ These areas

²⁰We use this relatively bigger DSGE model, instead of the PIM introduced in Section 2.8, to give the best chance to perfect information models to replicate the VAR impulse response functions. That model features a lot of mechanisms to fit the persistence in the data and has been found to fit the data well relatively to a Bayesian VAR model (Del Negro et al. 2007). Note that the Smets and Wouters’ model features serially

are computed by simulating 1 million parameter draws from the prior distribution described in Tables 1A and 1B in Smets and Wouters (2007).²¹ In Bayesian econometrics this analysis is called *prior predictive check* (Geweke 2005, p. 262; An and Schorfheide 2005; Del Negro and Schorfheide 2011; Leeper et al. 2011) and is often an important ex-ante specification check to assess whether a given model has any chance to replicate an empirical finding of interest. The rectangle with black solid edges marks the 70 percent posterior interquantile ranges for the contemporaneous response of the inflation expectations implied by the BVAR in Figure 1. This plot shows that the BVAR-implied contemporaneous responses lie far in the right tail of the prior distribution implied by the model developed by Smets and Wouters (2007). This means that for plausible parameterizations, the Smets and Wouters’ model cannot explain why the observed inflation expectations do not contemporaneously respond to monetary shocks. In particular, this structural model finds it very hard to rationalize contemporaneous drops in the one-quarter-ahead inflation expectations that are smaller than 10 basis points while the BVAR analysis suggests that the 70-percent lower-bound fall is about 6 basis points.

As far as the third fact is concerned, we compute the prior distribution for the half life of the response of inflation and inflation expectations to a monetary shock that raises the interest rate by 25 basis points in the model developed by Smets and Wouters (2007).²² The prior medians for the half life of the response of inflation, one-quarter-ahead inflation expectations, and four-quarter-ahead inflation expectations are five quarters, four quarters, and two quarters, respectively.²³ These numbers are way below what the VAR evidence suggests in Figure 1. The 90th percentile of these prior distributions is seven quarters for the response of inflation, six quarters for the response of one-quarter-ahead inflation expectations, and three quarters for the response of four-quarter-ahead inflation expectations. Such low percentiles suggest that the Smets and Wouters (2007) model is unable to adequately explain the large degree of persistence that characterizes the VAR response of inflation and inflation expectations to monetary innovations. In the next section, we will show that the DIM is significantly more successful at replicating this piece of VAR evidence.

3.5 Impulse Response Functions

In this section, we study the propagation of shocks in the estimated DIM. In Section 3.5.1, we analyze the propagation of monetary shocks. We deal with the transmission of non-policy shocks (i.e., demand shocks and aggregate technology shocks) in Section 3.5.2.

correlated monetary shocks as we assume in the DIM.

²¹We rescale the monetary shocks so that the contemporaneous response of the interest rate in the model is equal to 25 basis points, and we discard those prior draws that imply a drop in the interest rate immediately after a contractionary monetary shock.

²²Plots of these prior distributions are reported in Appendix H.

²³The prior means are very close to the prior medians.

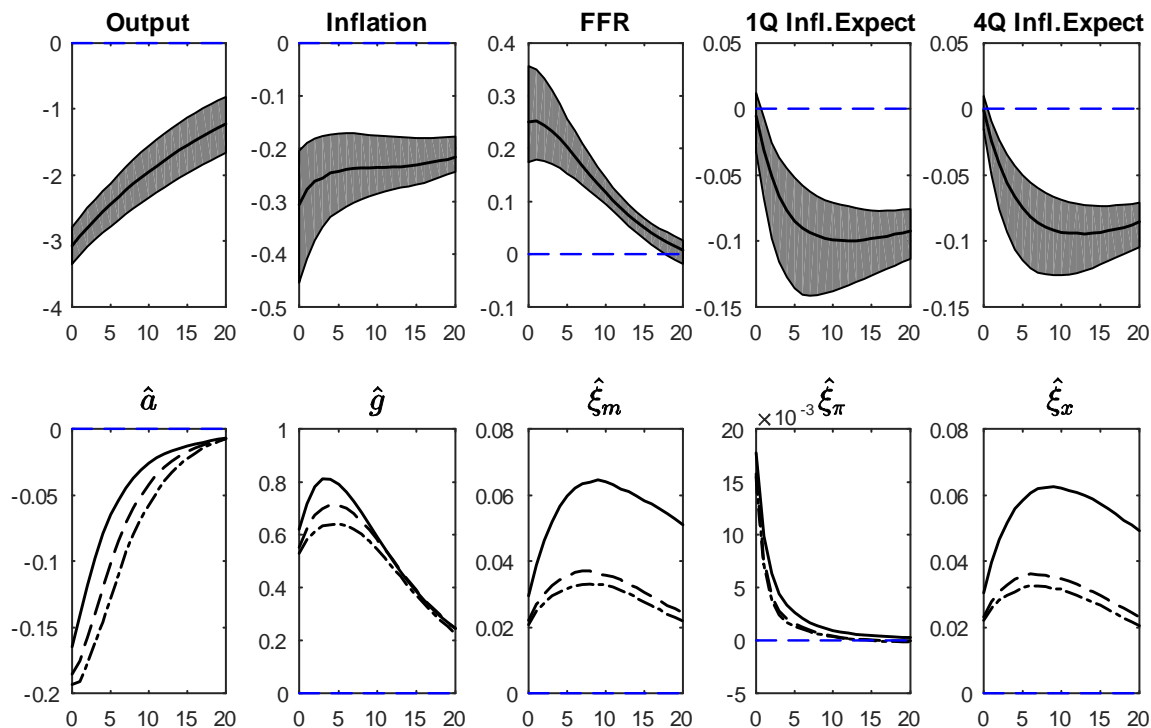


Figure 3: Impulse Response Functions to a Contractionary Monetary Shock. *Upper graphs:* Impulse response functions of output, inflation, the federal funds rate (FFR), and one- and four-quarter-ahead inflation expectations in percentage deviations from their steady-state values to a monetary shock that raises the federal funds rate by 25 bps. The responses of inflation, the federal funds rate, and inflation expectations are annualized. The solid lines denote posterior means. The gray areas denote 90 percent credible sets. The horizontal axis in all graphs measures the number of quarters after the shock. *Lower graphs:* Responses of average expectations about the five exogenous state variables in percentage deviations from their steady-state level. Black solid lines denote the average first-order expectations. Dashed black lines denote the average second-order expectations. Dashed-dotted lines denote the average third-order expectations.

3.5.1 Propagation of Monetary Shocks

Figure 3 shows the impulse response functions (and their 95 percent posterior credible sets in gray) of the level of real output (GDP), the inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a monetary shock that raises the interest rate by 25 basis points. Four features of these impulse response functions have to be emphasized. First, the DIM delivers impulse response functions of inflation expectations that look remarkably similar to those implied by the VAR model introduced in Section 3.4 (Figure 1). This similarity is striking if one takes into account that the DIM is a small-scale model. As discussed in Section 3.4, even a state-of-the-art perfect-information model, such as the one developed by Smets and Wouters (2007), has hard time explaining the high degree of persistence that characterizes the response of inflation and inflation expectations to monetary shocks implied by the VAR model. Second, the responses of inflation and inflation

expectations are very sluggish, even though the estimated degree of nominal rigidities is quite small. These persistent patterns are in line with the VAR evidence introduced in Section 3.4. Third, the DIM predicts fairly strong real effects of money. Sluggish adjustments in prices imply a lower path for households' inflation expectations. Consequently, the expected path of the real interest rate shifts upward after the contractionary monetary shock, leading the Euler equation (7) to predict a large drop in real activity.²⁴ Fourth, firms' inflation expectations respond positively to contractionary monetary shocks with some posterior probability, which is also consistent with the VAR evidence depicted in Figure 1.

In the lower graphs of Figure 3, we report the response of the average higher-order expectations (from the first order up to the third order). Notice that the signaling channel induces firms to partially believe that the rise in the interest rate is due to either a positive demand shock or a negative technology shock or an overestimation of the output gap by the central bank. These signaling effects are not surprising given the poor quality of the private signal about the demand conditions relative to the public signal and the information mix conveyed by the policy signal, as discussed in Section 3.2. Furthermore, note that the average expectations about the state of monetary policy $\hat{\xi}_{m,t}$ and those about the central bank's measurement error for the output gap $\hat{\xi}_{x,t}$ virtually respond in the same fashion to monetary shocks. The only thing firms observe about these two exogenous processes is the interest rate \hat{R}_t . Therefore, firms can rationally tell the two processes apart only if these processes have different statistical properties. For instance, firms understand that a persistent change in the interest rate is relatively more likely to be explained by the more persistent shock. Nevertheless, the statistical properties of these two exogenous processes turn out to be almost identical (Table 1.) Notice that, for given parameters of the monetary policy rule, the in-sample dynamics of these two exogenous processes and hence their statistical properties are exactly determined by the actual and real-time output gap as well as actual and real-time inflation, which are observable variables in our estimation.

Figure 3 shows that a contractionary monetary shock causes inflation expectations (i) to barely move upon impact and (ii) to remain persistently away from their steady-state value (disanchoring). In Section 3.4, we showed that these patterns are observed in the data. The DIM offers a structural interpretation of these patterns. To this end, we show the contribution of the average expectations $X_{t|t}^{(0:k)}$ about the five exogenous state variables to the response of inflation and inflation expectations in Figure 4. The vertical bars show the response of inflation (left graph) and inflation expectations (middle and right graphs) to a contractionary monetary shock obtained by simulating the DIM using only one of the five exogenous state variables and

²⁴It should be noted that the log-linearized Euler equation (7) can be expanded forward to obtain $\hat{x}_t = -\sum_{k=0}^{\infty} (\hat{R}_{t+k} - E_t \hat{\pi}_{t+k} - \hat{r}_{t+k}^n)$, where $(\hat{R}_t - E_t \hat{\pi}_t)$ denotes the real interest rate and \hat{r}_t^n denotes the natural rate, which is a function of aggregate technology shocks and demand shocks.

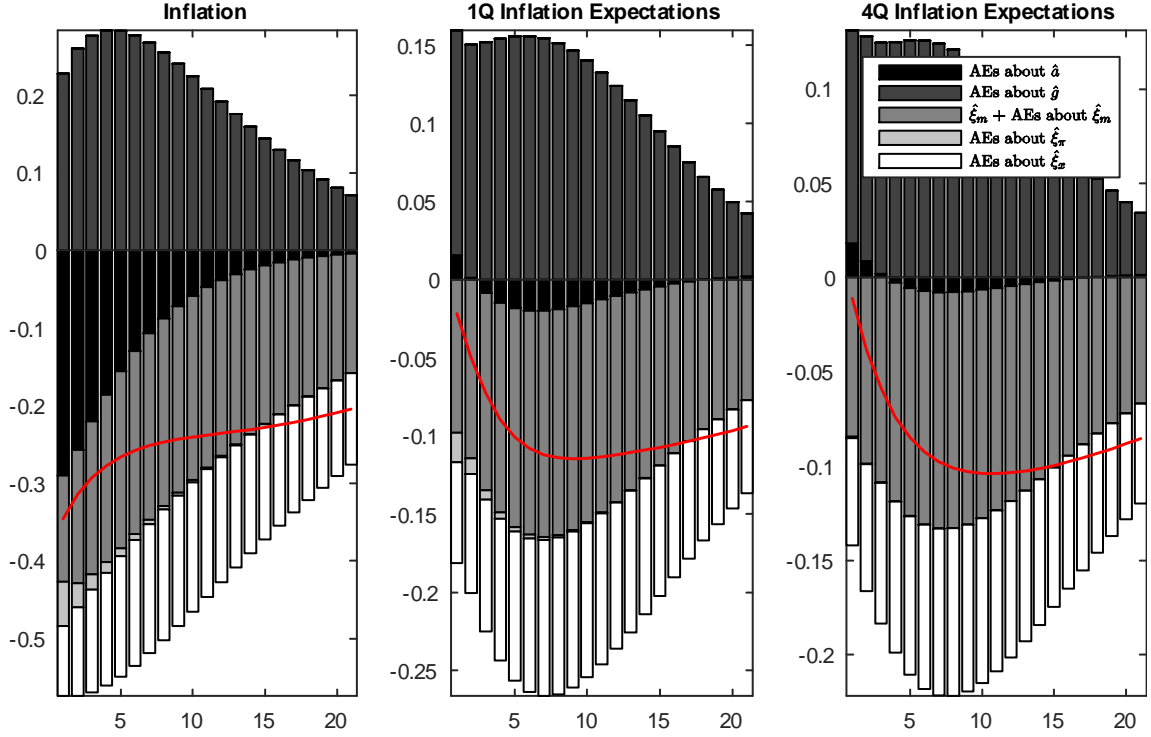


Figure 4: Contributions of average expectations to the impulse response functions of inflation and inflation expectations to a monetary shock that raises the interest rate by 25 bps. Parameter values are set equal to the posterior mean. The solid red line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph). The vertical bars capture the contribution of the actual shocks and the average expectations about the level of aggregate technology \hat{a}_t , the demand conditions \hat{g}_t , and the three types of deviations from the monetary rule $\hat{\xi}_{m,t}$, $\hat{\xi}_{\pi,t}$, and $\hat{\xi}_{x,t}$ to inflation and inflation expectations.

the associated average expectations. The sum of the five vertical bars equals the response of inflation and inflation expectations (i.e., the solid red line) evaluated at the posterior mean reported in Table 1. Upon impact, the contribution of the average expectations about positive demand shocks (the dark gray bars lying in positive territory) almost perfectly offsets the contribution of the average expectations about the exogenous deviations from the monetary policy rule (the light gray and white bars lying in negative territory). This finding implies that the monetary tightening owing to the policy shock immediately signals that the central bank is responding to a demand shock, exerting upward pressures on inflation expectations. These signaling effects explain why inflation expectations hardly move as the monetary shock hits and then evolve sluggishly. The persistent disanchoring of inflation expectations observed in the longer run is explained by the fact that the monetary tightening ends up signaling the central bank's persistent mistakes in measuring the output gap, as captured by the white bars lying in negative territory. These signaling effects raise the half life of the response of inflation

expectations in line with what is observed in the data.²⁵

The DIM seems to overstate the persistence of inflation forecast errors after a monetary policy shock compared with the predictions of the VAR model introduced in Section 3.4. In this respect, we should not forget that the DIM is a very small-scale model. While a more sophisticated version of the DIM could do better at reproducing the persistence of the conditional forecast errors implied by the VAR model, no perfect information model can for the reasons explained in Section 3.4.

Real effects of money in the estimated DIM are stronger than what the VAR literature typically finds. Nakamura and Steinsson (2015) use unexpected changes in interest rates over a 30 minute window surrounding scheduled Federal Reserve announcements to identify monetary policy shocks in a reduced-form model. These scholars find that the response of inflation is small and delayed. They use this evidence to estimate the key parameters of a workhorse perfect-information New Keynesian model and find that the implied real effects of money are quantitatively larger than what is usually found by the VAR literature. Adding consumption habits is likely to dampen the response of output in the DIM. Nevertheless, this extension would substantially complicate the solution of the DIM, preventing Bayesian estimation. Incomplete information on the side of households could also cause consumption and output to respond more sluggishly to monetary shocks. This extension is discussed in Section 4.

As shown in the lower graphs of Figure 3, some average expectations respond very sluggishly to monetary shocks. While these persistent adjustments are crucial for such a stylized DSGE model to deliver a degree of persistence in line with the data, they may also raise concerns about what may appear to be an implausibly long period for firms to learn the true value of the exogenous state variables. These concerns will be addressed in Section 4. Shocks to the central bank's forecast errors regarding the output gap $\varepsilon_{x,t}$ propagate across the macroeconomy almost identically to the monetary shocks $\varepsilon_{m,t}$, and hence, their analysis is omitted.²⁶

²⁵To understand why signaling an adverse technology shock has deflationary consequences (black bars), recall that the average expectations about the real marginal cost in the imperfect-common-knowledge Phillips curve (6) are given by $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \hat{a}_{t|t}^{(k-1)}$, $k \geq 1$. Note that shocks are orthogonal and hence $\partial \hat{a}_{t|t}^{(0)} / \partial \varepsilon_{m,t} = \partial \hat{a}_t / \partial \varepsilon_{m,t} = 0$. Since expecting an adverse technology shock leads firms to expect a fall in output ($\hat{y}_{t|t}^{(1)}$), the average first-order expectations about the real marginal costs $\widehat{mc}_{t|t}^{(1)} = \hat{y}_{t|t}^{(1)} - \hat{a}_t$ would fall, driving down inflation and inflation expectations. If this first-order effect ($\widehat{mc}_{t|t}^{(1)} = \hat{y}_{t|t}^{(1)} - \hat{a}_t$) dominates the higher-order effects ($\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \hat{a}_{t|t}^{(k-1)}$, $k \geq 2$), then expecting a negative technology shock will bring about deflationary pressures.

²⁶The propagation of real-time measurement errors regarding inflation is less interesting from the perspective of this paper and is omitted. The results are available upon request.

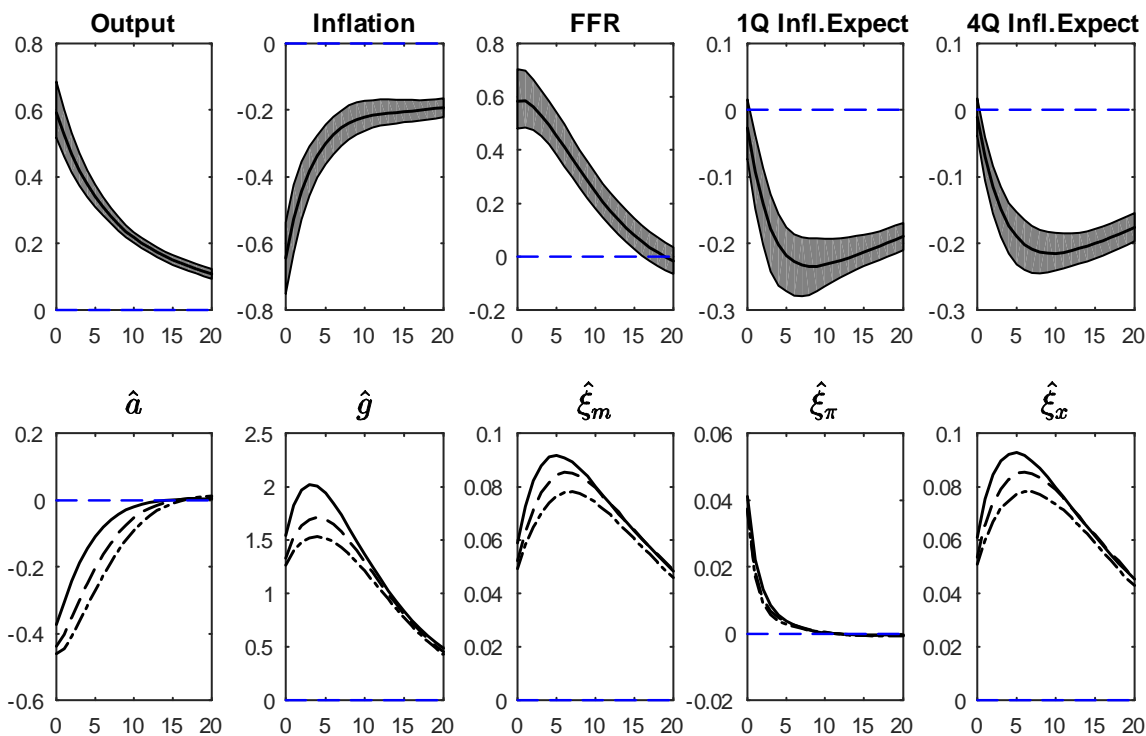


Figure 5: Impulse Response Functions to a Positive Demand Shock. *Upper graphs:* Impulse response functions of output, inflation, the federal funds rate (FFR), and one- and four-quarter-ahead inflation expectations in percentage deviations from their steady-state values to a one-standard deviation positive demand shock. The responses of inflation, the federal funds rate, and inflation expectations are annualized. The solid lines denote posterior means. The gray areas denote 90-percent credible sets. The horizontal axis in all graphs measures the number of quarters after the shock. *Lower graphs:* Responses of average expectations about the five exogenous state variables in percentage deviations from their steady-state level. Black solid lines denote the average first-order expectations. Dashed black lines denote the average second-order expectations. Dashed-dotted lines denote the average third-order expectations.

3.5.2 Propagation of Non-Policy Shocks

The propagation of a one-standard-deviation positive demand shock is described in Figure 5. This figure shows the responses of output in percentage deviations from its steady state. The responses of inflation, the federal funds rate, and inflation expectations are expressed in *annualized* percentage deviations from their steady-state value. Interestingly, inflation and inflation expectations respond *negatively* to demand shocks, while output responds positively. Note that the central bank raises its policy rate in the aftermath of a positive demand shock leading to two types of signaling effects. First, the monetary tightening induces firms to believe that a contractionary deviation from the monetary policy rule has happened. Second, the observed rise in the federal funds rate induces firms to believe that a negative technology

shock might have occurred. Figure 6 shows that both of these effects push inflation down,²⁷ countering the rise in inflation due to the positive demand shock, which is captured by the gray bars. While the second effect (captured by the black bars in Figure 6) has quantitatively a fairly small impact on inflation expectations, the first effect (captured by the white bars) appears to substantially contribute to pushing inflation expectations down. Furthermore, the second effect is generally shorter lived than the first one. The first effect is very persistent indeed, reflecting the following two facts. First, firms find it hard to disentangle whether changes in the policy rate are due to exogenous deviations from the monetary rule or are instead due to demand shocks for reasons that were analyzed in Section 3.2. Second, in the aftermath of a positive demand shock, monetary policy ends up signaling *persistent* contractionary deviations from the monetary rule. The high persistence of the exogenous state variables $\hat{\xi}_{m,t}$ and $\hat{\xi}_{x,t}$ in the estimated DIM (Table 1) clearly drives this result because rational firms know that when the central bank deviates from the rule, this behavior will last for a fairly long time.

Quite interestingly, the signaling channel transforms demand shocks ($\hat{\varepsilon}_{g,t}$) into supply shocks that move output and inflation in opposite directions. Unlike technology shocks (see Appendix I), this artificial supply shock implies a negative comovement between the federal funds rate and the rate of inflation, as well as between the interest rate and inflation expectations. This property is likely to help the model fit the 1970s, when the policy rate was relatively low while inflation and inflation expectations attained quite high values.

A drop in the policy rate owing to a positive technology shock induces firms to believe that the central bank is responding to either an expansionary deviation from the monetary policy rule ($\hat{\xi}_{m,t} < 0$ and $\hat{\xi}_{x,t} < 0$) or a negative demand shock.²⁸ These two signals turn out to have almost perfectly offsetting effects on inflation and inflation expectations. The fairly high accuracy of private information about aggregate technology clearly contributes to this result. Because of this, the propagation of technology shocks is qualitatively the same as that in perfect information models, with output responding positively and inflation responding negatively. An exact quantification of the signaling effects driven by technology shocks will be provided in the next section.

3.6 The Signaling Effects of Monetary Policy

In this section, we use the DIM to empirically assess the signaling effects of monetary policy on inflation and inflation expectations. To this end, we run a *Bayesian counterfactual experiment* using an algorithm that can be described as follows. In Step 1, for every posterior draw of the DIM parameters, we obtain the model’s predicted series for the five structural shocks (the

²⁷Signaling adverse technology shocks brings about deflationary pressures because it leads firms to anticipate a fall in output, as discussed in Section 3.5.1.

²⁸The propagation of technology shocks is analyzed in greater detail in Appendix I.

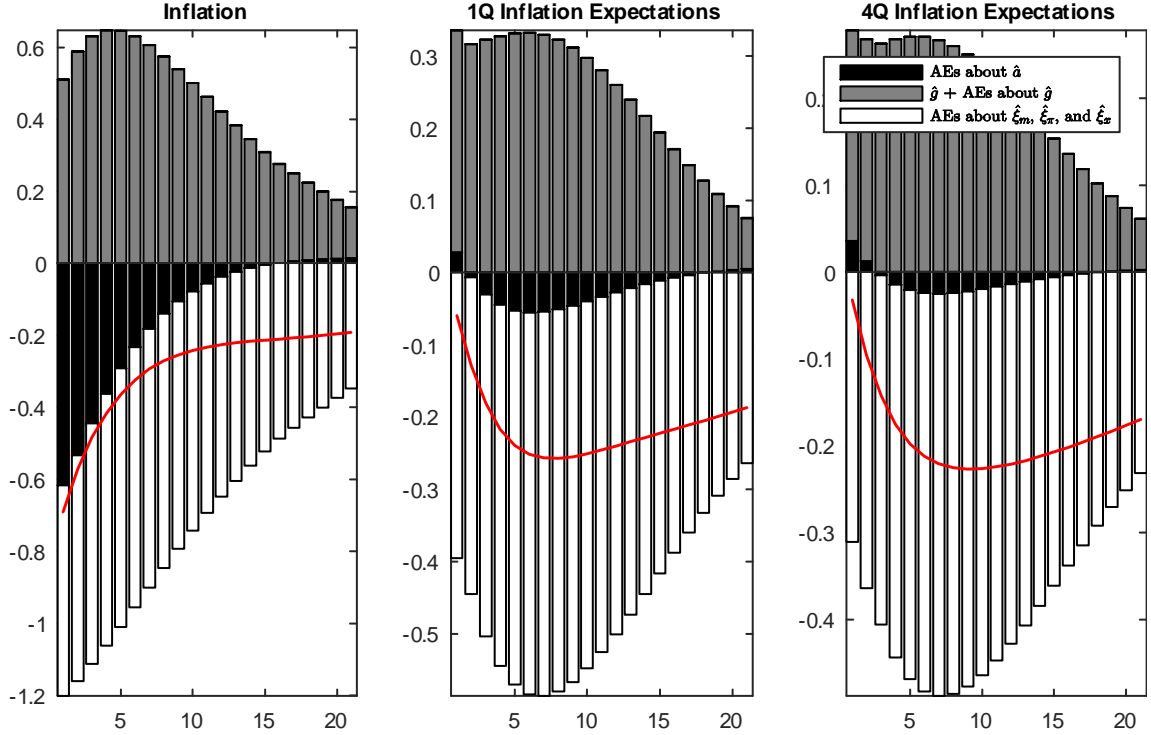


Figure 6: Contributions of average expectations to the impulse response functions of inflation and inflation expectations to a one-standard-deviation positive demand shock. Parameter values are set equal to the posterior mean. The solid red line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph). The vertical bars capture the contribution of the actual shocks and the average expectations about the level of aggregate technology \hat{a}_t , the demand conditions \hat{g}_t , and the three types of deviations from the monetary rule $\hat{\xi}_{m,t}$, $\hat{\xi}_{\pi,t}$, and $\hat{\xi}_{x,t}$ altogether to inflation and inflation expectations.

aggregate technology shock $\varepsilon_{a,t}$, the demand shock, $\varepsilon_{g,t}$, the monetary shock $\varepsilon_{m,t}$, and the shocks to the central bank’s measurement errors $\varepsilon_{\pi,t}$ and $\varepsilon_{x,t}$) using the two-sided Kalman filter and the seven observable variables introduced in Section 3.1. In Step 2, these filtered series of shocks are used to simulate the rate of inflation and inflation expectations from the following two models: (i) the DIM and (ii) the *counterfactual* DIM, in which monetary policy has *no signaling effects*. The latter model is obtained from the DIM by assuming that firms do not observe the history of the policy rate \hat{R}_t . This assumption implies that the signaling channel is inactive, so firms form their expectations by using only their private information (i.e., the history of the signals $\hat{a}_{j,t}$ and $\hat{g}_{j,t}$). In Step 3, we compute the mean of the simulated series across posterior draws for the two models.

The shocks are filtered in Step 1 by using the data set used for estimation and described in Section 3.1. Since this data set includes both the final (HP-filter-based) output gap and the real-time output gap from the Greenbook, the errors made by the central bank in measuring the current output gap $\hat{\xi}_{x,t}$ are identical to the errors measured by Orphanides (2004). This

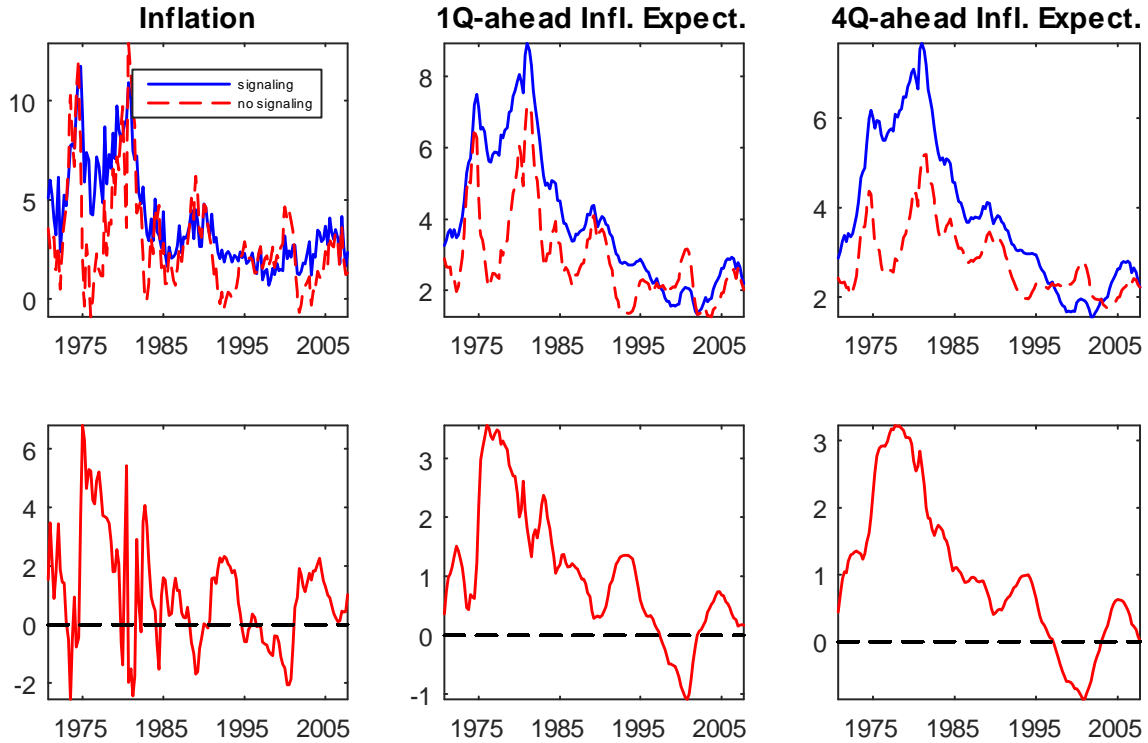


Figure 7: Signaling Effects of Monetary Policy on Inflation and Inflation Expectations. *Upper graphs:* Solid blue line: inflation rate (left graph) and inflation expectations (middle and right graphs) simulated from the estimated dispersed information model (DIM) using the two-sided filtered shocks from the estimated DIM. Red dashed line: simulated inflation (left plot) and inflation expectations (middle and right plot) from the counterfactual DIM, in which the signaling channel is shut down, using the two-sided filtered shocks from the estimated DIM. The vertical axis in all graphs measures units of percentage points of annualized rates. *Lower graphs:* The signaling effects of monetary policy on the annualized rate of inflation (left graph) and inflation expectations (middle and right graphs) in percent.

feature allows us to evaluate the signaling effects of monetary policy after controlling for the inflationary effects due to the Federal Reserve’s persistent mismeasurement of the output gap in the 1970s, which has been advocated by Orphanides (2001, 2002, 2003) to be one of the leading reasons for why inflation was so heightened in that decade.²⁹

The solid blue line in the upper graphs of Figure 7 denotes the inflation rate (left graph) and the inflation expectations (middle and right graphs) simulated from the DIM using the two-sided filtered shocks from the estimated DIM.³⁰ The red dashed line denotes the series of inflation (left graph) and inflation expectations (middle and right graphs) simulated from the counterfactual DIM, in which the signaling channel is shut down. The vertical difference

²⁹The series of the real-time output gap used for estimation is shown in Appendix F.

³⁰The simulated series of inflation is by construction the same as in the data. The simulated series of inflation expectations do not exactly replicate the actual data because of the measurement errors we attribute to the observed inflation expectations. However, the discrepancy between these two series is rather minuscule, since *i.i.d.* measurement errors just end up smoothing out the simulated series slightly.

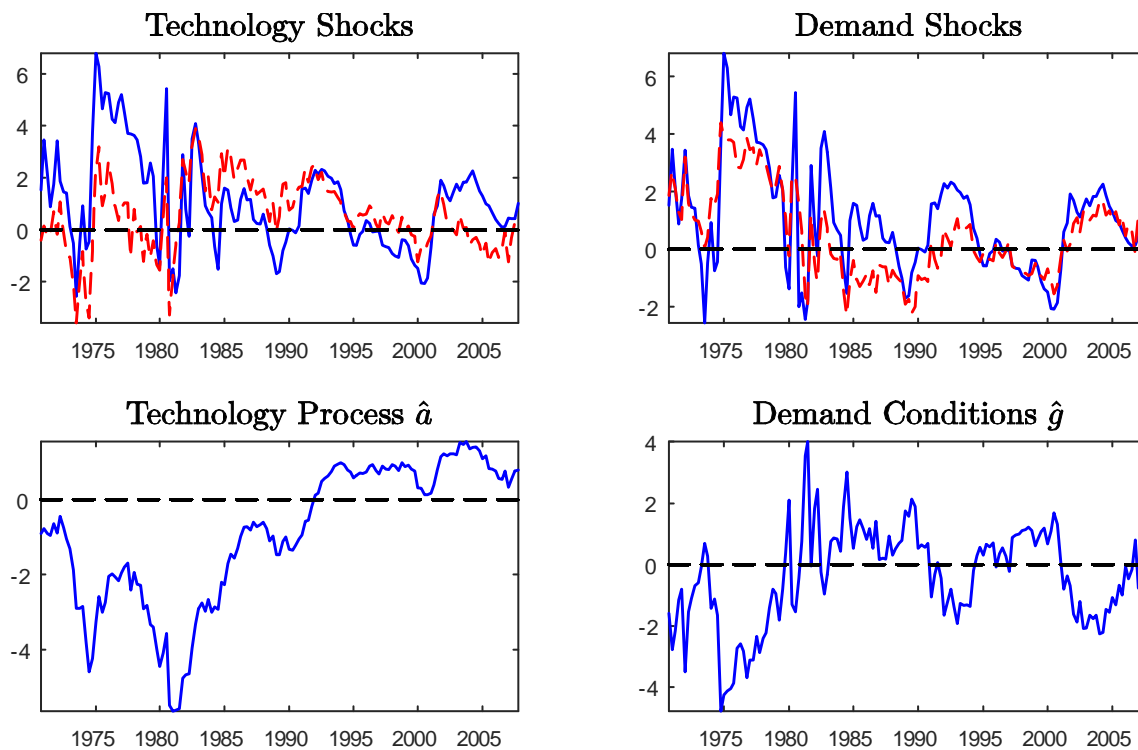


Figure 8: Contributions of Shocks to Signaling Effects on Inflation *Upper graphs*: Solid blue line: signaling effects of monetary policy on inflation. Red dashed line: signaling effects on inflation when only one type of shocks is used to simulate the DIM. *Left graphs*: only aggregate technology shocks are used. *Right graphs*: only demand shocks are used. *Lower graphs*: the two-sided filtered dynamics of aggregate technology \hat{a}_t (left) and demand conditions \hat{g}_t (right) used for simulation. All numbers are annualized and in percent.

between the two simulated series in the upper graphs captures the signaling effects of monetary policy over the sample period and is reported in the lower graphs.

In the model, signaling effects on inflation are particularly strong in the 1970s, adding up to 6.4 percentage points to the rate of inflation in that decade. Specifically, signaling effects play an important role in explaining why inflation was *persistently* heightened in the second half of the 1970s. These effects are even more pronounced when one looks at the signaling effects of monetary policy on inflation expectations. Signaling effects on inflation expectations are always positive until the end of the 1990s, largely explaining why in the data the inflation expectations were almost always above the rate of inflation from 1981:Q2 through the end of the 1980s.³¹

To shed light on the origin of the estimated signaling effects on inflation, in Figure 8 we compare the dynamics of the signaling effects on inflation (the solid blue line) with the signaling effects (the red dashed line) that are driven only by technology shocks (upper left graph) and

³¹In that period, observed one-quarter-ahead (four-quarter-ahead) inflation expectations have been 70 basis points (40 basis points) higher on average than the inflation rate.

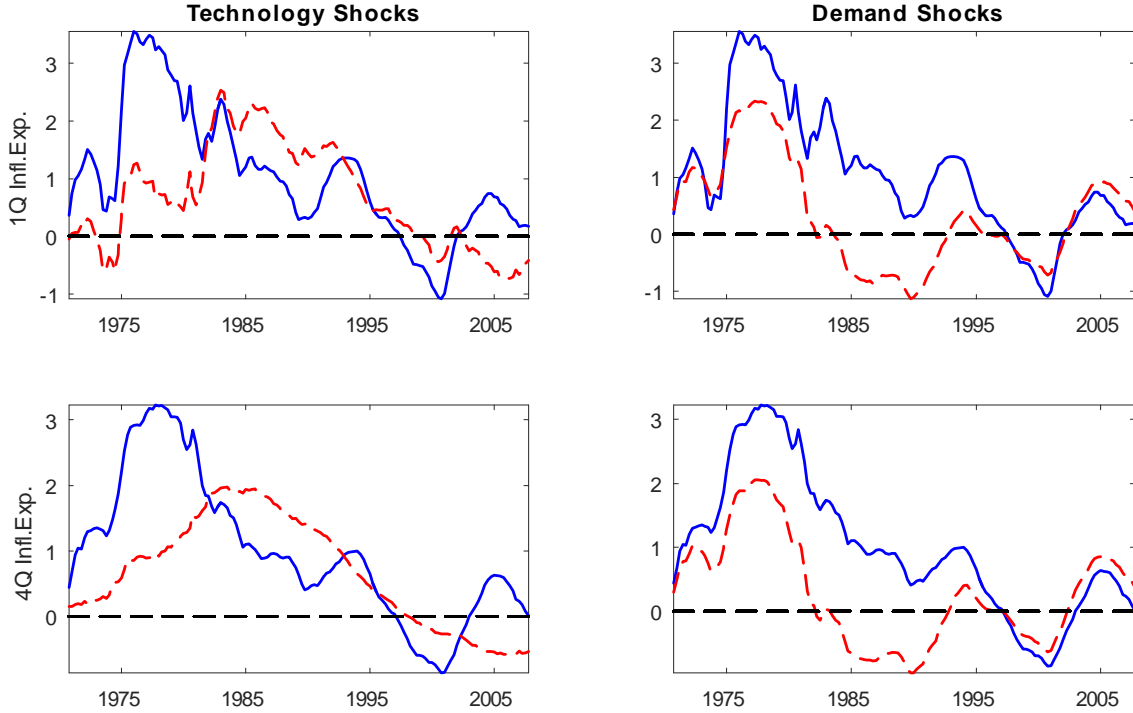


Figure 9: Contributions of Shocks to Signaling Effects on Inflation Expectations. Solid blue line: signaling effects of monetary policy on inflation expectations. Red dashed line: signaling effects on inflation expectations when only one shock is used to simulate the DIM. *Left graphs*: only aggregate technology shocks are used. *Right graphs*: only demand shocks are used. *Upper graphs*: One-quarter-ahead inflation expectations. *Lower graphs*: Four-quarter-ahead inflation expectations.

only by demand shocks (upper right graph).³² In the lower graphs of Figure 8, we show the two-sided filtered series of the two exogenous state variables \hat{a}_t (left graph) and \hat{g}_t (right graph) obtained in Step 1 of the Bayesian counterfactual experiment. We observe that most of the signaling effects on inflation in the 1970s are due to negative demand shocks because the signaling effects only driven by these shocks (the red dashed line) are similar to the overall signaling effects (the blue solid line) in that decade. In particular, two large negative demand shocks that occurred in 1974 explain the large and positive signaling effects on inflation in the second half of the 1970s. As shown in Section 3.5.2, negative demand shocks prompted the Federal Reserve to lower the policy rate, which signaled both persistent expansionary monetary shocks and long-lasting nowcast errors in measuring the output gap by the policymaker. In Section 4, we will show that there is strong VAR evidence supporting the realization of these two large demand shocks in 1974 once the signaling effects of monetary policy are taken into account for identifying these shocks.

³²These counterfactual series are obtained by simulating the estimated DIM by using only the two-sided filtered estimate of technology and demand shocks. The larger figure reporting the contribution to signaling effects of all the five shocks is available upon request. The omitted shocks are found to contribute only marginally to signaling effects of monetary policy on inflation and inflation expectations.

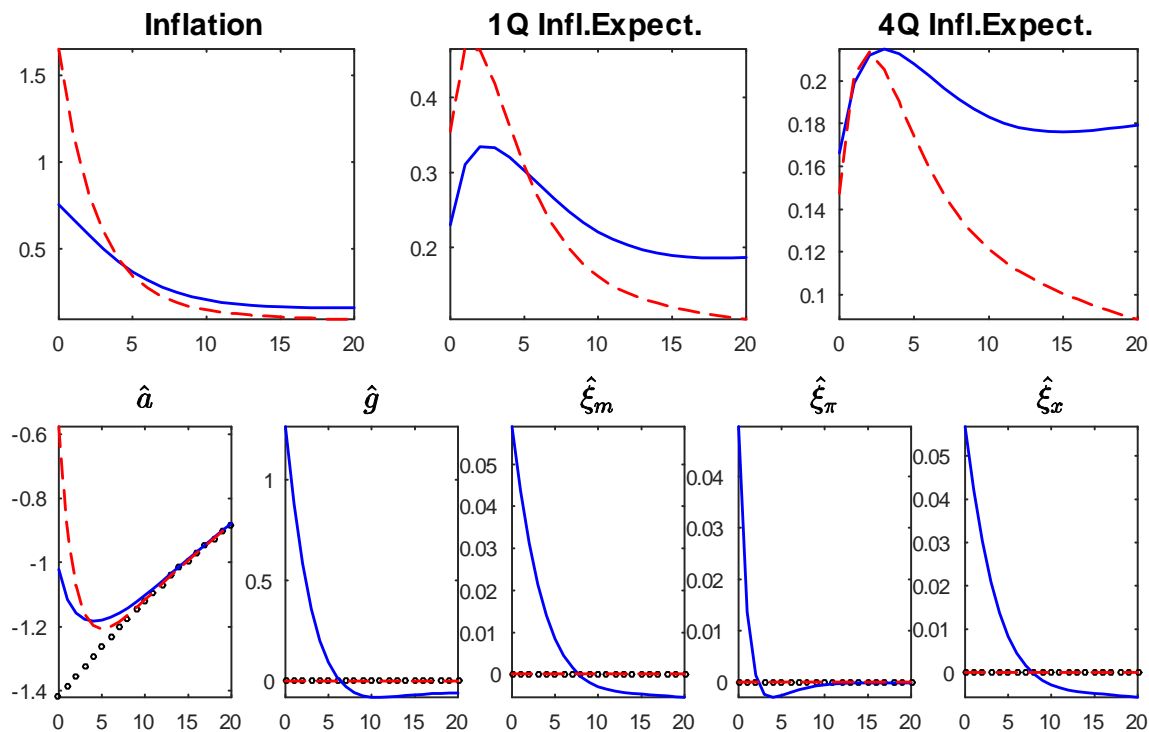


Figure 10: Signaling Effects on Inflation and Inflation Expectations Associated with a One-Standard-Deviation Negative Technology Shock. *Upper graphs:* Response of inflation (the left graph), one-quarter-ahead inflation expectations (the middle graph), and four-quarter-ahead inflation expectations (the right graph) in the estimated DIM, with signaling effects (the solid blue line) and in the counterfactual DIM, with no signaling effects (the red dashed line). *Lower graphs:* Black circles denote the response of the five exogenous state variables to a negative technology shock. The solid blue line denotes the response of the average first-order expectations about the five exogenous states in the estimated DIM, with signaling effects. The red dashed line denotes the response of the average first-order expectations about the five exogenous states in the counterfactual DIM, with no signaling effects.

Signaling effects associated with positive technology shocks contributed to raising inflation by up to 3 percentage points in 1975-1976. However, this contribution was quite short-lived because of the predominance of negative technology shocks in the 1970s, which brought about deflationary signaling effects, as shown in the upper left graph of Figure 8. According to the model, in the 1980s and in the early 1990s, the signaling effects of monetary policy on inflation are predominantly driven by aggregate technology shocks. Improvements in aggregate technology during this period induced the Federal Reserve to carry out a monetary policy that ended up signaling expansionary deviations from the monetary policy rule.

The main drivers of signaling effects on *inflation expectations* are shown in Figure 9. These graphs compare the dynamics of the signaling effects on inflation expectations (blue solid line) with the technology-driven (left graphs) and the demand-driven (right graphs) signaling effects on the one-quarter-ahead (upper graphs) and four-quarter-ahead (lower graphs) inflation expectations, which are denoted with the red dashed line. Similar to the signaling effects on

inflation, the signaling effects on inflation expectations during the 1970s are largely driven by demand shocks (see the right plots). The red dashed line in the left graphs of Figure 9 shows that technology-driven signaling effects on inflation expectations started building up slowly in the 1970s, which was a decade characterized by large and repeated negative technology shocks. This slow-moving pattern suggests that technology shocks bring about *delayed* signaling effects on inflation expectations. This pattern is fairly different from the dynamics that characterized the technology-driven signaling effects on inflation, which move around the zero line during the 1970s in the upper left graph of Figure 8. The improvements in aggregate technology observed from 1982 through the early 1990s slowly bring about a downward trend in the technology-driven signaling effects on inflation expectations. However, these effects are delayed and signaling effects on inflation expectations remain positive until the mid-1990s. This largely explains why inflation expectations were higher on average than inflation throughout the 1980s.

Why do negative technology shocks raise inflation expectations through the signaling channel with delays? To investigate this question, in Figure 10 we show the response of inflation (the upper left graph) and inflation expectations (the upper middle and right graphs) to a one-standard deviation negative technology shock in the estimated DIM (the solid blue line) and in the counterfactual DIM with no signaling effects (the red dashed line). The difference between these two lines captures the signaling effects due to negative technology shocks. Two features deserve to be emphasized. First, while signaling effects associated with technology shocks predominantly affect inflation at short horizons, inflation expectations are primarily influenced at longer horizons. Second, signaling effects on inflation and inflation expectations switch in sign and become inflationary a few quarters past the shock. This happens because six quarters after a negative technology shock, firms consider the policy rate to be lower than the level that they would have expected based on their beliefs about inflation and the output gap. Consequently, monetary policy starts signaling long-lasting expansionary deviations from the monetary rule ($\hat{\xi}_{m,t} < 0$ and $\hat{\xi}_{x,t} < 0$), as shown in the lower graphs of Figure 10. This suggests that large negative technology shocks that occurred in the late 1970s and early 1980s brought about signaling effects of monetary policy that contributed to slowly raising inflation expectations well into the 1980s. Conversely, improvements in technological conditions throughout the 1980s caused signaling effects on inflation expectations to slowly fall from mid-1980s through the end of the 1990s, as shown in the left graphs of Figure 9.

4 Discussion

The sluggish dynamics of beliefs in the DIM seem to be quite successful in explaining the persistent macroeconomic dynamics of inflation and inflation expectations. However, one may argue that such persistent dynamics of beliefs imply that firms are implausibly confused about the

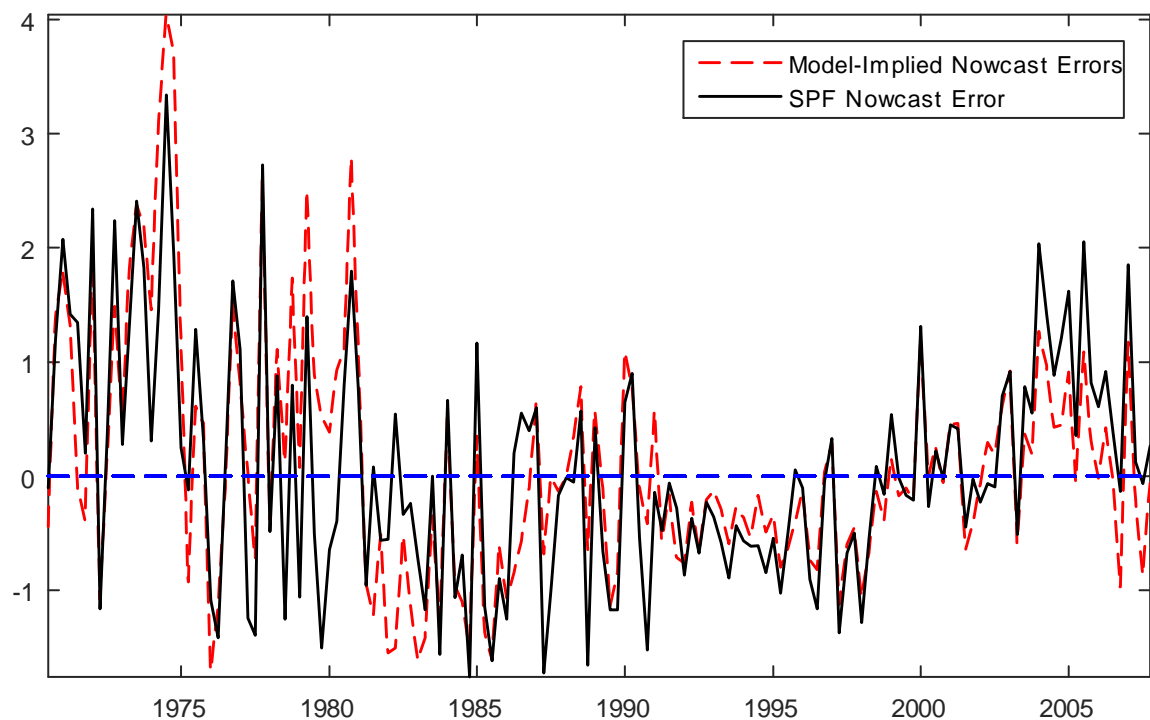


Figure 11: Inflation Nowcast Errors. The model-implied nowcast errors are obtained by subtracting the smoothed estimates of firms’ inflation nowcast (i.e., $\ln\pi_* + \hat{\pi}_{t|t}^{(0)}$) from the realized inflation rate. Smoothed estimates are obtained by setting the value of the DIM parameters to their posterior mean. Nowcast errors are reported in percentage points of annualized rates.

aggregate state of the economy. To mitigate this concern, we have included one-quarter-ahead and four-quarter-ahead inflation expectations in our data set for estimation. In addition, an important check to assess the plausibility of the information set is to compare the nowcast errors for inflation predicted by the DIM ($\hat{\pi}_t - \hat{\pi}_{t|t}^{(1)}$) to those measured by the *Survey of Professional Forecasters*. Figure 11 shows this comparison. The two nowcast errors exhibit a great deal of comovement with a correlation coefficient of 0.82. Furthermore, the mean of the absolute nowcast errors for inflation is 0.79 in the model vis-a-vis 0.81 in the data. This result suggests that the degree of information incompleteness in the estimated DIM is not implausible. It should also be noted that perfect information models predict that nowcasts errors are counterfactually equal to zero.

We showed that the signaling channel explains the heightened inflation and inflation expectations observed in the 1970s because of two large negative demand shocks that occurred in 1974. These two shocks caused the Federal Reserve to lower the policy rate, signaling expansionary monetary shocks and central bank’s mismeasurement of the output gap and inflation. Were there negative demand shocks in 1974? Recall that the signaling channel mutes the propagation of demand shocks so that they look like supply shocks, moving output and inflation in opposite directions (Figure 5). We know that *traditional* demand shocks were not so impor-

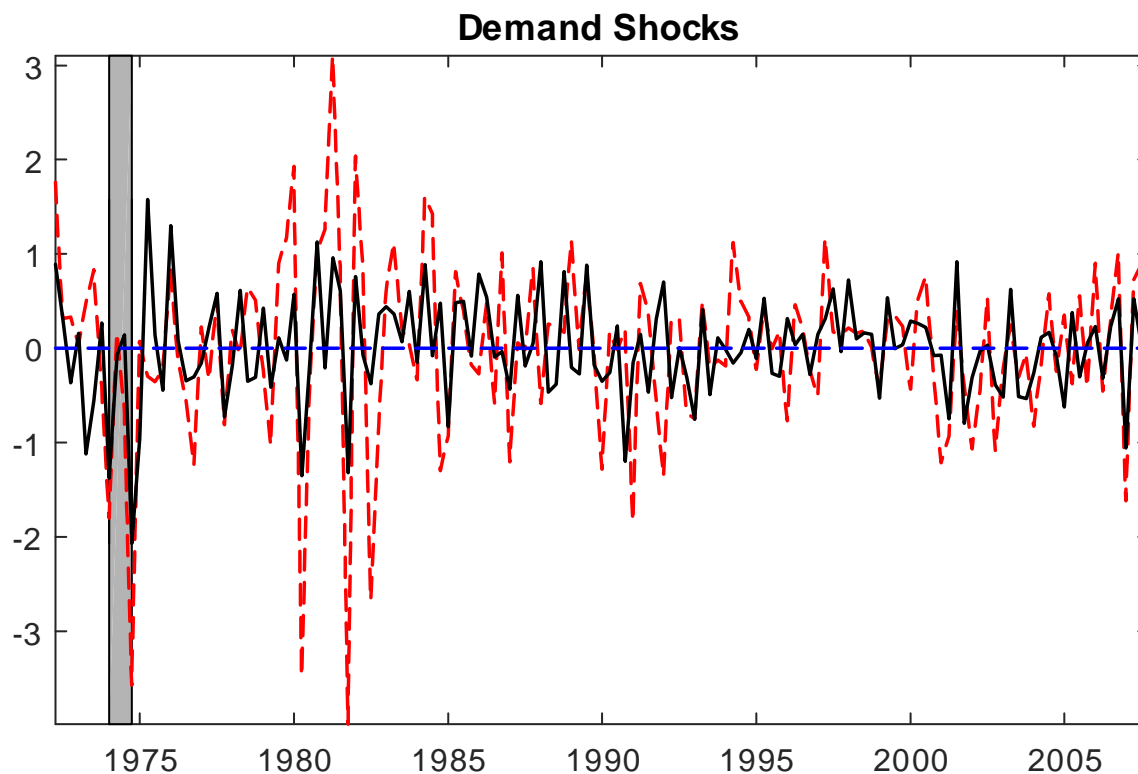


Figure 12: Posterior median of the implied demand shocks identified by applying sign restrictions to the BVAR model. The solid black line denotes the (posterior median of the) demand shocks identified using the VAR model. The red dashed line captures the (posterior median of the) smoothed estimates of demand shocks implied by the dispersed information model.

tant in the 1970s. But what about demand shocks *after controlling for the signaling effects of monetary policy*? Is there any evidence that these demand shocks *in disguise* actually occurred in the 1970s and, more specifically, in 1974? The answer to this question is yes. To reach this conclusion, we estimate a Bayesian VAR model using a large data set that includes GDP growth, consumption growth, investment growth, the growth rate of real compensation per hour, the growth rate of money (M2), the federal funds rate, the inflation rate, the growth rate of labor productivity, one-quarter-ahead (SPF) inflation expectations, and four-quarter-ahead (SPF) inflation expectations. Apart from the last two observables, this data set is very similar to the one used in the influential study by Christiano, Eichenbaum and Evans (2005).

Demand shocks are identified by using sign restrictions that are consistent with the DIM (Figure 5); that is, positive demand shocks are assumed to lower inflation and to raise output as well as the federal funds rate for the first five quarters in a row. Figure 12 shows the historical sequence of demand shocks identified by applying these sign restrictions to the VAR (solid black line) along with the two-sided estimate of these innovations identified using the estimated DIM (red dashed line). First of all, the two series seem to be quite positively correlated, with the VAR estimates being less volatile. The coefficient of correlation is slightly below 0.60.

Second, judging from the series of the demand innovations implied by the VAR model (i.e., the red dashed line), it looks like the first half of the 1970s was characterized by a few of these large negative demand shocks in disguise. Third, consistent with the estimated DIM, the VAR model suggests that the two largest negative demand shocks of the sample were realized in 1974, which is marked by the gray area in the plot. According to the estimated DIM, these two large negative demand shocks gave rise to sizable signaling effects on inflation and inflation expectations throughout the second half of the 1970s.

On the narrative side, 1974 was a year of high political uncertainty in the U.S. because of the unraveling of the so-called Watergate scandal, which led the House of Representatives to open the impeachment process against President Richard Nixon that year. The scandal started in 1972 but it arguably became a major constitutional crisis starting on February 6, 1974, when the House of Representatives approved a resolution giving the Judiciary Committee authority to investigate impeachment of the President.³³ On July 27, 1974, the House Judiciary Committee voted to recommend the first article of impeachment against the President: obstruction of justice. The House recommended the second article, abuse of power, on July 29, 1974. The next day, on July 30, 1974, the House recommended the third article: contempt of Congress. On August 9, 1974, President Richard Nixon resigned. These events undoubtedly marked a period of high political uncertainty for U.S. households that might have well had an impact on how they discounted future events.

Another concern has to do with the assumption that firms observe only one endogenous variable, the interest rate, and all the remaining private information comes from exogenous signals. As discussed in Section 2.6, our information structure is built on the imperfect-common-knowledge literature (Woodford 2002; Adam 2007; Nimark 2008). However, one may be reasonably concerned that firms are not allowed, for instance, to use information about the quantities they sell for price-setting decisions. The log-linear approximation to Equation (2) implies that observing the quantities sold would be one additional endogenous signal that would perfectly reveal nominal output to firms. We find that estimating a DIM in which firms pay attention to nominal output would deliver a lower marginal likelihood ($-586.76 < -319.89$), suggesting that this alternative specification of the DIM fits the data rather poorly. Allowing firms to perfectly observe nominal output ends up endowing them with too much information, critically weakening the ability of the dispersed information model to generate macroeconomic fluctuations with the right degree of persistence. This is particularly true for the case of the federal funds rate and for the observed inflation expectations. This empirical shortcoming of the DIM in which firms observe nominal output cannot be fixed by simply dropping the exogenous signals $a_{j,t}$ and $g_{j,t}$ from firms' information set. This finding suggests that firms may not pay attention to nominal

³³Even though many resolutions to impeach the President were submitted in 1972 and 1973, the Judiciary Committee had always refused to take up the case.

output when making their price-setting decisions, even though information about this variable is arguably quite cheap to obtain. This result is in line with the empirical study by Andrade et al. (2014), who use the *Blue Chip Financial Forecasts* to document that disagreement about inflation and GDP is quite high at short horizons.

5 Concluding Remarks

This paper studies a DSGE model in which information is dispersed across price setters and the interest rate set by the central bank has signaling effects. In this model, monetary impulses propagate through two channels: (i) the channel based on the central bank’s ability to affect the real interest rate due to price stickiness and dispersed information and (ii) the signaling channel. The latter arises because changing the policy rate conveys information about the central bank’s assessment of inflation and the output gap to price setters.

We fit the model to a data set that includes the *Survey of Professional Forecasters* as a measure of price setters’ inflation expectations. We perform an econometric evaluation of the model with signaling effects of monetary policy, showing that this model can closely replicate the response of inflation expectations to monetary shocks implied by a VAR model. We also find that the signaling channel makes demand shocks look like supply shocks that move inflation and output in opposite directions. Moreover, we show that signaling effects of monetary policy can account for (i) why inflation and inflation expectations were so persistently heightened in the 1970s³⁴ and (ii) why inflation expectations fell more sluggishly than inflation after the famous disinflationary policy carried out by the Federal Reserve in early 1980s.

While there exist several channels through which central banks can communicate with markets nowadays, our paper focuses on interest-rate-based communication. Interest-rate-based communication was virtually the only form of central bank’s communication until February 1994 in the U.S. (Campbell et al., 2012). The importance of this type of communication has been growing in recent years. See, for instance, the widespread endorsement of the practice of providing information about the likely future path of the policy rate, which goes by the name of *forward guidance*. While we do not study the effects of forward guidance in this paper, we have shown how to formalize interest-rate-based communication in dynamic general equilibrium models and how to use these models to formally evaluate the macroeconomic effects of this type of communication.

³⁴Other popular theories for why inflation rose in the 1970s are (i) the bad luck view (e.g., Cogley and Sargent 2005; Sims and Zha 2006; Primiceri 2005; and Liu, Waggoner, and Zha 2011), (ii) the lack of commitment view (e.g., Chari, Christiano, and Eichenbaum 1998; Christiano and Gust 2000), (iii) the policy mistakes view (e.g., Sargent 2001; Clarida, Galí, and Gertler 2000; Lubik and Schorfheide 2004; Primiceri 2006; Coibion and Gorodnichenko 2011), and (iv) fiscal and monetary interactions view (e.g., Sargent, Williams, and Zha 2006; Bianchi and Ilut 2012; Bianchi and Melosi, 2014).

Changes in the Federal Reserve's attitude toward inflation stabilization have been documented by Davig and Leeper (2007), Justiniano and Primiceri (2008), Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010) and Bianchi (2013). Time-varying model parameters allow us to study how signaling effects of monetary policy on the macroeconomy have changed over time. This fascinating topic is left for future research.

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Technical Appendix (Not For Publication)

The appendices are organized as follows. In Appendix A, we derive of the imperfect-common-knowledge Phillips curve (6). Appendix B details an algorithm to solve the dispersed information model. In Appendix C, we characterize the transition equations for the average higher-order expectations about the exogenous state variables – that is, equation (10) in the main text. In Appendix D, we characterize the laws of motion for the three endogenous state variables (i.e., inflation $\hat{\pi}_t$, real output \hat{y}_t and the interest rate \hat{R}_t). In Appendix E, we define a set of measures to quantify the amount of information conveyed by the signals observed by firms. The measurement equations used for estimating the dispersed information model and the perfect information model are in Appendix F. In Appendix G, we conduct a posterior predictive analysis on the ability of the DIM to fit the persistence in the data. Appendix H shows the prior distribution for the half life of the response of inflation and inflation expectations to monetary shocks implied by the Bayesian VAR studied in Section 3.4. In Appendix I, we show the propagation of technology shocks in the model with the signaling channel. In Appendix J, we show the signaling effects of monetary policy on inflation and inflation expectations in the aftermath of a demand shock. The case of imperfectly informed households is discussed in Appendix K.

A The Imperfect-Common-Knowledge Phillips Curve

The log-linear approximation to the labor supply can be given by $\hat{c}_t = \hat{w}_t$. Recalling that the resource constraint implies that $\hat{y}_t = \hat{c}_t$, we can then rewrite the labor supply as follows:

$$\hat{y}_t = \hat{w}_t. \tag{11}$$

Log-linearizing the equation for the real marginal costs yields

$$\widehat{mc}_{j,t} = \hat{w}_t - \hat{a}_t - \varepsilon_{j,t}^a.$$

We can then write

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{w}_{j,t} - \hat{a}_t - \varepsilon_{j,t}^a,$$

where $\mathbb{E}_{j,t}$ is the expectations conditioned on firm j 's information set at time t ($\mathcal{I}_{j,t}$) defined in (5). Using equation (11) for replacing \hat{w}_t yields

$$\mathbb{E}_{j,t}\widehat{mc}_{j,t} = \mathbb{E}_{j,t}\hat{y}_t - \hat{a}_t - \varepsilon_{j,t}^a.$$

By integrating across firms, we obtain the average expectations on marginal costs:

$$\widehat{mc}_{t|t}^{(1)} = \hat{y}_{t|t}^{(1)} - \hat{a}_t.$$

The linearized price index can be written as

$$\int \hat{p}_{j,t}^* dj = \frac{\theta}{1-\theta} \hat{\pi}_t.$$

Recall that we defined $\hat{p}_{j,t}^* = \ln P_{j,t}^* - \ln P_t$ and $\hat{\pi}_t = \ln P_t - \ln P_{t-1} - \ln \pi_*$. After some algebraic manipulation, we write

$$\ln P_t = \theta (\ln P_{t-1} + \ln \pi_*) + (1-\theta) \int (\ln P_{j,t}^*) dj. \quad (12)$$

The price-setting problem leads to the following first-order conditions:

$$\mathbb{E} \left[\sum_{s=0}^{\infty} (\beta\theta)^s \frac{\xi_{j,t+s}}{P_{t+s}} \left[(1-\nu) \pi_*^s + \nu \frac{MC_{j,t+s}}{P_{j,t}^*} \right] y_{j,t+s} | \mathcal{I}_{j,t} \right] = 0.$$

We define the variables:

$$p_{j,t}^* = \frac{P_{j,t}^*}{P_t}; \quad mc_{j,t} = \frac{MC_{j,t}}{P_t}.$$

And then we write

$$\mathbb{E} \left\{ \xi_{j,t} \left[1 - \nu + \nu \frac{mc_{j,t}}{p_{j,t}^*} \right] y_{j,t} + \sum_{s=1}^{\infty} (\beta\theta)^s \xi_{j,t+s} \left[(1-\nu) \pi_*^s + \nu \frac{mc_{j,t+s}}{p_{j,t}^*} (\prod_{\tau=1}^s \pi_{t+\tau}) \right] y_{j,t+s} | \mathcal{I}_{j,t} \right\} = 0. \quad (13)$$

First realize that the terms inside the square brackets are equal to zero at the steady state, and hence, we do not care about the terms outside them. We can write

$$\mathbb{E} \left[\left[1 - \nu + \nu mc_{j,*} e^{\widehat{mc}_{j,t} - \hat{p}_{j,t}^*} \right] + \sum_{s=1}^{\infty} (\beta\theta)^s \left[(1-\nu) \pi_*^s + \nu mc_{j,*} e^{\widehat{mc}_{j,t+s} - \hat{p}_{j,t}^* + \sum_{\tau=1}^s \hat{\pi}_{t+\tau}} \right] | \mathcal{I}_{j,t} \right] = 0.$$

Taking the derivatives yields

$$\mathbb{E} \left[\widehat{mc}_{j,t} - \hat{p}_{j,t}^* + \sum_{s=1}^{\infty} (\beta\theta)^s \left[\left(\widehat{mc}_{j,t+s} - \hat{p}_{j,t}^* + \sum_{\tau=1}^s \hat{\pi}_{t+\tau} \right) \right] | \mathcal{I}_{j,t} \right] = 0.$$

We can take the term $\hat{p}_{j,t}^*$ out of the sum operator in the second term and gather the common

term to obtain

$$\mathbb{E} \left[\widehat{m}c_{j,t} - \frac{1}{1-\beta\theta} \widehat{p}_{j,t}^* + \sum_{s=1}^{\infty} (\beta\theta)^s \left(\widehat{m}c_{j,t+s} + \sum_{\tau=1}^s \widehat{\pi}_{t+\tau} \right) | \mathcal{I}_{j,t} \right] = 0.$$

Recall that $\widehat{p}_{j,t}^* \equiv \ln P_{j,t}^* - \ln P_t$ and cannot be taken out of the expectation operator. We write

$$\ln P_{j,t}^* = (1-\beta\theta) \mathbb{E} \left[\widehat{m}c_{j,t} + \frac{1}{1-\beta\theta} \ln P_t + \sum_{s=1}^{\infty} (\beta\theta)^s \left(\widehat{m}c_{j,t+s} + \sum_{\tau=1}^s \widehat{\pi}_{t+\tau} \right) | \mathcal{I}_{j,t} \right]. \quad (14)$$

Rolling this equation one step ahead yields

$$\ln P_{j,t+1}^* = (1-\beta\theta) \mathbb{E} \left[\widehat{m}c_{j,t+1} + \frac{1}{1-\beta\theta} \ln P_{t+1} + \sum_{s=1}^{\infty} (\beta\theta)^s \left(\widehat{m}c_{j,t+s+1} + \sum_{\tau=1}^s \widehat{\pi}_{t+\tau+1} \right) | \mathcal{I}_{j,t+1} \right].$$

Taking firm j 's conditional expectation at time t on both sides and applying the law of iterated expectations, we obtain the following:

$$\mathbb{E} (\ln P_{j,t+1}^* | \mathcal{I}_{j,t}) = (1-\beta\theta) \mathbb{E} \left[\widehat{m}c_{j,t+1} + \frac{1}{1-\beta\theta} \ln P_{t+1} + \sum_{s=1}^{\infty} (\beta\theta)^s \left(\widehat{m}c_{j,t+s+1} + \sum_{\tau=1}^s \widehat{\pi}_{t+\tau+1} \right) | \mathcal{I}_{j,t} \right].$$

We can take $\widehat{m}c_{j,t+1}$ inside the sum operator and write

$$\mathbb{E} (\ln P_{j,t+1}^* | \mathcal{I}_{j,t}) = (1-\beta\theta) \mathbb{E} \left[\frac{1}{1-\beta\theta} \ln P_{t+1} + \frac{1}{\beta\theta} \sum_{s=1}^{\infty} (\beta\theta)^s \widehat{m}c_{j,t+s} + \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \widehat{\pi}_{t+\tau+1} | \mathcal{I}_{j,t} \right].$$

Therefore,

$$\sum_{s=1}^{\infty} (\beta\theta)^s \mathbb{E} [\widehat{m}c_{j,t+s} | \mathcal{I}_{j,t}] = \frac{\beta\theta}{1-\beta\theta} [\mathbb{E} (\ln P_{j,t+1}^* | \mathcal{I}_{j,t}) - \mathbb{E} (\ln P_{t+1} | \mathcal{I}_{j,t})] - \beta\theta \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E} [\widehat{\pi}_{t+\tau+1} | \mathcal{I}_{j,t}]. \quad (15)$$

Hence, the equation (14) can be rewritten as:

$$\begin{aligned} \ln P_{j,t}^* &= (1-\beta\theta) \left\{ \mathbb{E} [\widehat{m}c_{j,t} | \mathcal{I}_{j,t}] + \frac{1}{1-\beta\theta} \mathbb{E} [\ln P_t | \mathcal{I}_{j,t}] + \sum_{s=1}^{\infty} (\beta\theta)^s \mathbb{E} [\widehat{m}c_{j,t+s} | \mathcal{I}_{j,t}] \right\} \\ &\quad + (1-\beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E} [\widehat{\pi}_{t+\tau} | \mathcal{I}_{j,t}]. \end{aligned}$$

By substituting the result in equation (15), we obtain

$$\begin{aligned}
\ln P_{j,t}^* &= (1 - \beta\theta) \left[\mathbb{E}[\widehat{m}c_{j,t}|\mathcal{I}_{j,t}] + \frac{1}{1 - \beta\theta} \mathbb{E}[\ln P_t|\mathcal{I}_{j,t}] \right] \\
&\quad + \beta\theta \left[\mathbb{E}(\ln P_{j,t+1}^*|\mathcal{I}_{j,t}) - \mathbb{E}(\ln P_{t+1}|\mathcal{I}_{j,t}) \right] - (1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^{s+1} \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}] \\
&\quad + (1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau}|\mathcal{I}_{j,t}].
\end{aligned}$$

We consider the last term and write

$$\begin{aligned}
(1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau}|\mathcal{I}_{j,t}] &= (1 - \beta\theta) \beta\theta \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}] + (1 - \beta\theta) \sum_{s=2}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau}|\mathcal{I}_{j,t}] \\
&= (1 - \beta\theta) \beta\theta \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}] + \\
&\quad + (1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^{s+1} \left(\sum_{\tau=1}^s [\mathbb{E}[\widehat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}]] + \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}] \right).
\end{aligned}$$

It then follows that

$$\begin{aligned}
(1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau}|\mathcal{I}_{j,t}] &= (1 - \beta\theta) \beta\theta \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}] \\
&\quad + (1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^{s+1} \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}] \\
&\quad + (1 - \beta\theta) \left(\sum_{s=1}^{\infty} (\beta\theta)^{s+1} \right) \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}].
\end{aligned}$$

Because $(\sum_{s=1}^{\infty} (\beta\theta)^{s+1}) = \frac{(\beta\theta)^2}{1 - \beta\theta}$, then after simplifying, we can write that

$$\begin{aligned}
(1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^s \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau}|\mathcal{I}_{j,t}] &= \beta\theta \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}] \\
&\quad + (1 - \beta\theta) \sum_{s=1}^{\infty} (\beta\theta)^{s+1} \sum_{\tau=1}^s \mathbb{E}[\widehat{\pi}_{t+\tau+1}|\mathcal{I}_{j,t}].
\end{aligned}$$

We substitute this result into the original equation to get the following expression:

$$\begin{aligned}
\ln P_{j,t}^* &= (1 - \beta\theta) \mathbb{E}[\widehat{m}c_{j,t}|\mathcal{I}_{j,t}] + \mathbb{E}[\ln P_t|\mathcal{I}_{j,t}] \\
&\quad + \beta\theta \left[\mathbb{E}(\ln P_{j,t+1}^*|\mathcal{I}_{j,t}) + \mathbb{E}[\widehat{\pi}_{t+1}|\mathcal{I}_{j,t}] - \mathbb{E}(\ln P_{t+1}|\mathcal{I}_{j,t}) \right]. \tag{16}
\end{aligned}$$

Note that by definition $\hat{\pi}_{t+1} \equiv \ln P_{t+1} - \ln P_t - \ln \pi_*$. Hence, we can write

$$\begin{aligned} \ln P_{j,t}^* &= (1 - \beta\theta) \cdot \mathbb{E}[\widehat{mc}_{j,t} | \mathcal{I}_{j,t}] + (1 - \beta\theta) \mathbb{E}[\ln P_t | \mathcal{I}_{j,t}] \\ &\quad + \beta\theta \cdot \mathbb{E}(\ln P_{j,t+1}^* | \mathcal{I}_{j,t}) - \beta\theta \ln \pi_*. \end{aligned} \quad (17)$$

We denote firm j 's average k -th order expectation about an arbitrary variable \hat{x}_t as

$$\mathbb{E}^{(k)}(\hat{x}_t | \mathcal{I}_{j,t}) \equiv \int \mathbb{E} \left(\int \mathbb{E} \left(\dots \left(\int \mathbb{E}(\hat{x}_t | \mathcal{I}_{j,t}) dj \right) \dots | \mathcal{I}_{j,t} \right) dj | \mathcal{I}_{j,t} \right) dj,$$

where expectations and integration across firms are taken k times.

Let us denote the average reset price as $\ln P_t^* = \int \ln P_{j,t}^* dj$. Note that we can rewrite equation (12) as follows

$$\ln P_t = \theta (\ln P_{t-1} + \ln \pi_*) + (1 - \theta) \ln P_t^*. \quad (18)$$

Furthermore, we can integrate equation (17) across firms to obtain an equation for the average reset price:

$$\begin{aligned} \ln P_t^* &= (1 - \beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1 - \beta\theta) \ln P_{t|t}^{(1)} \\ &\quad + \beta\theta \ln P_{t+1|t}^{*(1)} - \beta\theta \ln \pi_*, \end{aligned} \quad (19)$$

where $x_{t|t}^{(1)}$ denotes the average first-order expectations about an arbitrary variable x_t of the model (e.g., the real marginal costs).

Let us plug equation (19) into equation (18) as follows:

$$\begin{aligned} \ln P_t &= \theta \ln P_{t-1} + (\theta - (1 - \theta)\beta\theta) \ln \pi_* \\ &\quad + (1 - \theta) \left[(1 - \beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1 - \beta\theta) \ln P_{t|t}^{(1)} + \beta\theta \ln P_{t+1|t}^{*(1)} \right]. \end{aligned} \quad (20)$$

From the fact that $\ln P_t = \hat{\pi}_t + \ln P_{t-1} + \ln \pi_*$ and from the price index (12), we get the

following:³⁵

$$\ln P_{t+1}^* = \frac{\hat{\pi}_{t+1}}{1-\theta} + \ln P_t + \ln \pi_*.$$

Furthermore, the following fact is easy to establish:

$$\ln P_{t+1} = \hat{\pi}_{t+1} + \ln P_t + \ln \pi_*.$$

Applying these three results to equation (20) yields

$$\begin{aligned} \hat{\pi}_t + \ln P_{t-1} + \ln \pi_* &= \theta \ln P_{t-1} + (\theta - (1-\theta)\beta\theta) \ln \pi_* \\ &+ (1-\theta) \left[(1-\beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1-\beta\theta) \ln P_{t|t}^{(1)} + \beta\theta \left(\frac{\hat{\pi}_{t+1|t}^{(1)}}{1-\theta} + \ln P_{t|t}^{(1)} + \ln \pi_* \right) \right]. \end{aligned} \quad (21)$$

Algebraic manipulations yield the following equation:³⁶

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta) \cdot \widehat{mc}_{t|t}^{(1)} + (1-\theta)\hat{\pi}_{t|t}^{(1)} + \beta\theta \left(\hat{\pi}_{t+1|t}^{(1)} \right). \quad (22)$$

By repeatedly taking firm j 's expectation and then averaging the resulting equation across firms, we get

$$\hat{\pi}_{t|t}^{(k)} = (1-\theta)(1-\beta\theta) \cdot \widehat{mc}_{t|t}^{(k+1)} + (1-\theta)\hat{\pi}_{t|t}^{(k+1)} + \beta\theta \left(\hat{\pi}_{t+1|t}^{(k+1)} \right).$$

Repeatedly substituting these equations for $k \geq 1$ back in equation (22) yields the imperfect-common-knowledge Phillips curve:

$$\hat{\pi}_t = (1-\theta)(1-\beta\theta) \sum_{k=1}^{\infty} (1-\theta)^{k-1} \widehat{mc}_{t|t}^{(k)} + \beta\theta \sum_{k=1}^{\infty} (1-\theta)^{k-1} \hat{\pi}_{t+1|t}^{(k)}.$$

³⁵This last result comes from observing that

$$\ln P_t = \theta (\ln P_{t-1} + \ln \pi_*) + (1-\theta) \ln P_t^*.$$

By using the fact that $\ln P_t = \hat{\pi}_t + \ln P_{t-1} + \ln \pi_*$:

$$\hat{\pi}_t + \ln P_{t-1} + \ln \pi_* = \theta (\ln P_{t-1} + \ln \pi_*) + (1-\theta) \ln P_t^*.$$

Rolling one period forward, we get

$$\hat{\pi}_{t+1} = (\theta - 1) (\ln P_t + \ln \pi_*) + (1-\theta) \ln P_{t+1}^*.$$

And finally, by rearranging the terms, we get the result in the text.

³⁶We disregard the terms capturing firms' higher-order beliefs about last period's price level. The role of these terms are quantitatively negligible in the estimated model, while they would substantially raise the computational burden when solving the model.

B Solving the Dispersed Information Model

We solve the model assuming common knowledge of rationality (Nimark 2008) and focusing on equilibria where the higher-order expectations about the exogenous state variables (that is, $X_{t|t}^{(0:k)} \equiv \left[\widehat{a}_{t|t}^{(s)}, \widehat{g}_{t|t}^{(s)}, \widehat{\xi}_{m,t|t}^{(s)}, \widehat{\xi}_{\pi,t|t}^{(s)}, \widehat{\xi}_{x,t|t}^{(s)} : 0 \leq s \leq k \right]'$) follow the VAR(1) process in equation (10). Note that we truncate the order of the average expectations at $k < \infty$. As shown in Appendix D, for a given vector \mathbf{v}_0 such that $\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$, the structural equations of the model can be written in the following linear form:

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \quad (23)$$

where \mathbb{E}_t denotes the expectation operator conditional on a complete information set (i.e., an information set that includes the history of all structural shocks).

For a given parameter set Θ_{DIM} , take the following steps:

Step 0 Set $i = 1$ and guess the matrices $\mathbf{M}^{(i)}$, $\mathbf{N}^{(i)}$, and $\mathbf{v}_0^{(i)}$.

Step 1 Set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$ and solve the model given by equation (10) and equation (23) through a standard linear rational expectations model solver (e.g., Blanchard and Kahn 1980; Sims 2002). The solver delivers the matrix $\mathbf{v}_0^{(i+1)}$ such that $\mathbf{s}_t = \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)}$. As we will show in Appendix D, the matrices Γ_0 , Γ_1 , and Γ_2 in equation (23) are functions of the model parameter Θ_{DIM} as well as the guessed matrices $\mathbf{M}^{(i)}$ and $\mathbf{v}_0^{(i)}$.

Step 2 Given the law of motion (10) for $X_{t|t}^{(0:k)}$, in which we set $\mathbf{M} = \mathbf{M}^{(i)}$ and $\mathbf{N} = \mathbf{N}^{(i)}$, equation $\widehat{a}_{j,t} = \widehat{a}_t + \widetilde{\sigma}_a \varepsilon_{j,t}^a$ for the signal concerning the aggregate technology, equation $\widehat{g}_{j,t} = \widehat{g}_t + \widetilde{\sigma}_g \varepsilon_{j,t}^g$ for the signal concerning the demand conditions, and the equation

$$\widehat{R}_t = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \mathbf{v}_0^{(i+1)} X_{t|t}^{(0:k)}$$

for the endogenous policy signal $\widehat{R}_t \in \mathbf{s}_t$ solve the firms' signal extraction problem through the Kalman filter and determine the matrices $\mathbf{M}^{(i+1)}$ and $\mathbf{N}^{(i+1)}$. Appendix C provides a detailed explanation of how we characterize these matrices.

Step 3 If $\|\mathbf{M}^{(i)} - \mathbf{M}^{(i+1)}\| < \varepsilon_m$, $\|\mathbf{N}^{(i)} - \mathbf{N}^{(i+1)}\| < \varepsilon_n$, and $\|\mathbf{v}_0^{(i)} - \mathbf{v}_0^{(i+1)}\| < \varepsilon_v$ for any $\varepsilon_m > 0$, $\varepsilon_n > 0$, and $\varepsilon_v > 0$ and small, STOP or else set $i=i+1$ and go to STEP 1.

Given equation (10) and equation $\mathbf{s}_t = \mathbf{v}_0^{(i)} X_{t|t}^{(0:k)}$ obtained in step 1, the law of motion of the model variables is as follows:

$$\begin{bmatrix} X_{t|t}^{(0:k)} \\ \mathbf{s}_t \end{bmatrix} = \begin{bmatrix} \mathbf{M}^{(i+1)} & \mathbf{0} \\ \mathbf{v}_0^{(i+1)} \mathbf{M}^{(i+1)} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{t-1|t-1}^{(0:k)} \\ \mathbf{s}_{t-1} \end{bmatrix} + \begin{bmatrix} \mathbf{N}^{(i+1)} \\ \mathbf{v}_0^{(i+1)} \mathbf{N}^{(i+1)} \end{bmatrix} \boldsymbol{\varepsilon}_t. \quad (24)$$

C Transition Equation of High-Order Expectations

In this section, we show how to derive the law of motion of the average higher-order expectations of the exogenous state variables (i.e., $\widehat{a}_t, \widehat{g}_t, \widehat{\xi}_{m,t}, \widehat{\xi}_{\pi,t}, \widehat{\xi}_{x,t}$) for given parameter values and the matrix of coefficients \mathbf{v}_0 . We focus on equilibria where the average expectations evolve according to

$$X_{t|t}^{(0:k)} = \mathbf{M}X_{t-1|t-1}^{(0:k)} + \mathbf{N}\boldsymbol{\varepsilon}_t, \quad (25)$$

where $\boldsymbol{\varepsilon}_t \equiv \begin{bmatrix} \varepsilon_{a,t} & \varepsilon_{g,t} & \varepsilon_{m,t} & \varepsilon_{\pi,t} & \varepsilon_{x,t} \end{bmatrix}'$. We set $\mathbf{X}_t \equiv X_{t|t}^{(0:k)}$, for notational convenience. Firms' reduced-form state-space model can be concisely cast as follows:

$$\mathbf{X}_t = \mathbf{M}\mathbf{X}_{t-1} + \mathbf{N}\boldsymbol{\varepsilon}_t, \quad (26)$$

$$\mathbf{Z}_t(j) = \mathbf{D}\mathbf{X}_t + \mathbf{Q}e_{j,t}, \quad (27)$$

where

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_1 & \mathbf{d}_2 & (\mathbf{1}_3^T \mathbf{v}_0) \end{bmatrix}',$$

with $\mathbf{d}'_1 = [1, \mathbf{0}_{1 \times 5(k+1)-1}]$, $\mathbf{d}'_2 = [0, 1, \mathbf{0}_{1 \times 5k+3}]$, $\mathbf{1}_3^T = [0, 0, 1]$, and $e_{j,t} = [\varepsilon_{j,t}^a, \varepsilon_{j,t}^g]'$ and

$$\mathbf{Q} = \begin{bmatrix} \tilde{\sigma}_a & 0 \\ 0 & \tilde{\sigma}_g \\ 0 & 0 \end{bmatrix}.$$

Solving the firms' signal extraction problem requires applying the Kalman filter. The Kalman equation pins down firm j 's first-order expectations about the model's state variables $\mathbf{X}_{t|t}(j)$ and the associated conditional covariance matrix $\mathbf{P}_{t|t}$:

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1} [\mathbf{Z}_t(j) - \mathbf{Z}_{t|t-1}(j)], \quad (28)$$

$$\mathbf{P}_{t|t} = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{D}'\mathbf{F}_{t|t-1}^{-1}\mathbf{D}\mathbf{P}'_{t|t-1}, \quad (29)$$

where

$$\mathbf{P}_{t|t-1} = \mathbf{M}\mathbf{P}_{t-1|t-1}\mathbf{M}' + \mathbf{N}\mathbf{N}', \quad (30)$$

and the matrix $\mathbf{F}_{t|t-1} \equiv E[\mathbf{Z}_t\mathbf{Z}'_t|\mathbf{Z}^{t-1}]$, which can be shown to be

$$\mathbf{F}_{t|t-1} = \mathbf{D}\mathbf{P}_{t|t-1}\mathbf{D}' + \mathbf{Q}\mathbf{Q}'. \quad (31)$$

Therefore, combining equation (29) with equation (30) yields

$$\mathbf{P}_{t+1|t} = \mathbf{M} \left[\mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1} \mathbf{D}' \mathbf{F}_{t|t-1}^{-1} \mathbf{D} \mathbf{P}'_{t|t-1} \right] \mathbf{M}' + \mathbf{N} \mathbf{N}'. \quad (32)$$

Define the Kalman-gain matrix as $\mathbf{K}_t \equiv \mathbf{P}_{t|t-1} \mathbf{D}' \mathbf{F}_{t|t-1}^{-1}$. Write the law of motion of firm j 's first-order beliefs about \mathbf{X}_t as

$$\mathbf{X}_{t|t}(j) = \mathbf{X}_{t|t-1}(j) + \mathbf{K}_t [\mathbf{D} \mathbf{X}_t + \mathbf{Q} e_{j,t} - \mathbf{D} \mathbf{X}_{t|t-1}(j)],$$

where we have combined equations (28) and (27). By recalling that $\mathbf{X}_{t|t-1}(j) = \mathbf{M} \mathbf{X}_{t-1|t-1}(j)$, we obtain

$$\mathbf{X}_{t|t}(j) = (\mathbf{M} - \mathbf{K} \mathbf{D} \mathbf{M}) \mathbf{X}_{t-1|t-1}(j) + \mathbf{K} [\mathbf{D} \mathbf{M} \cdot \mathbf{X}_{t-1} + \mathbf{D} \mathbf{N} \cdot \boldsymbol{\varepsilon}_t + \mathbf{Q} e_{j,t}]. \quad (33)$$

The vector $\mathbf{X}_{t|t}(j)$ contains firm j 's first-order expectations about the model's state variables. Integrating across firms yields the law of motion of the average expectation about \mathbf{X}_t :

$$\mathbf{X}_{t|t}^{(1)} = (\mathbf{M} - \mathbf{K} \mathbf{D} \mathbf{M}) \mathbf{X}_{t-1|t-1}^{(1)} + \mathbf{K} [\mathbf{D} \mathbf{M} \cdot \mathbf{X}_{t-1} + \mathbf{D} \mathbf{N} \cdot \boldsymbol{\varepsilon}_t].$$

Note that $X_{t|t}^{(0:\infty)} = [X_t, X_{t|t}^{(1:\infty)}]'$ and that

$$X_t = \underbrace{\begin{bmatrix} \rho_a & 0 & 0 & 0 & 0 & \mathbf{0} \\ 0 & \rho_g & 0 & 0 & 0 & \mathbf{0} \\ 0 & 0 & \rho_m & 0 & 0 & \mathbf{0} \\ 0 & 0 & 0 & \rho_\pi & 0 & \mathbf{0} \\ 0 & 0 & 0 & 0 & \rho_x & \mathbf{0} \end{bmatrix}}_{\mathbf{R}_1} X_{t-1|t-1}^{(0:k)} + \underbrace{\begin{bmatrix} \sigma_a & 0 & 0 & 0 & 0 \\ 0 & \sigma_g & 0 & 0 & 0 \\ 0 & 0 & \sigma_m & 0 & 0 \\ 0 & 0 & 0 & \sigma_\pi & 0 \\ 0 & 0 & 0 & 0 & \sigma_x \end{bmatrix}}_{\mathbf{R}_2} \cdot \boldsymbol{\varepsilon}_t.$$

So by using the assumption of common knowledge in rationality, we can fully characterize the matrices \mathbf{M} and \mathbf{N} :

$$\mathbf{M} = \begin{bmatrix} \mathbf{R}_1 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{5 \times 5} & \mathbf{0}_{5 \times 5k} \\ \mathbf{0}_{5k \times 5} & (\mathbf{M} - \mathbf{K} \mathbf{D} \mathbf{M})|_{(1:5k, 1:5k)} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K} (\mathbf{D} \mathbf{M})|_{(1:5k, 1:5(k+1))} \end{bmatrix}, \quad (34)$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{R}_2 \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{K} \mathbf{D} \mathbf{N}|_{(1:5k, 1:5)} \end{bmatrix}, \quad (35)$$

where $\cdot|_{(n_1:n_2, m_1:m_2)}$ denotes the submatrix obtained by taking the elements lying between the n_1 -th row and the n_2 -th row and between the m_1 -th column and the m_2 -th column. Note that \mathbf{K} in equation (34) and equation (35) denotes the steady-state Kalman gain matrix, which is

obtained by iterating the equations (30) and (32) until convergence.

D The Laws of Motion for the Endogenous State Variables

In this section we introduce some useful results and characterize the law of motion (23) for the endogenous state variables, which are inflation $\hat{\pi}_t$, real output \hat{y}_t , and the (nominal) interest rate \hat{R}_t .

D.1 Preliminaries

The *assumption of common knowledge in rationality* ensures that agents use the actual law of motion of higher-order expectations to forecast the dynamics of the higher-order expectations. The following propositions turn out to be useful for what follows:

Proposition 1 $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$ for any $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_t = \mathbf{v}_0 X_{t|t}^{(0:k)}$. Then common knowledge in rationality implies that $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 X_{t|t}^{(s:k+s)}$. ■

Since we truncate beliefs after the k -th order, we define the matrix $\mathbf{T}^{(s)}$ as follows:

$$\mathbf{T}^{(s)} \equiv \begin{bmatrix} \mathbf{0}_{5(k-s+1) \times 5s} & \mathbf{I}_{5(k-s+1)} \\ \mathbf{0}_{5s \times 5s} & \mathbf{0}_{5s \times (k+1-s)5} \end{bmatrix},$$

and we approximate the law of motion for $\mathbf{s}_{t|t}^{(s)}$ as $\mathbf{s}_{t|t}^{(s)} = \mathbf{v}_0 \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

Proposition 2 *The following holds true:* $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ for any $0 \leq s \leq k$.

Proof. We conjectured that $\mathbf{s}_{t+h} = \mathbf{v}_0 X_{t+h|t}^{(0:k)}$. Given equation (10), it follows that $\mathbf{s}_{t+h} = \mathbf{v}_0 \left(\mathbf{M}^h X_{t|t}^{(0:k)} + \mathbf{N} \boldsymbol{\varepsilon}_{t+1} \right)$. Common knowledge in rationality implies that repeatedly taking firms' expectations and then averaging across firms leads to an expression for the law of motion of the average higher-order expectations: $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h X_{t|t}^{(s:k+1)}$ for any s . ■

Since we truncate beliefs after the k -th order, we can approximate the law of motion for the average higher-order expectations as $\mathbf{s}_{t+h|t}^{(s)} = \mathbf{v}_0 \mathbf{M}^h \mathbf{T}^{(s)} X_{t|t}^{(0:k)}$ for any $0 \leq s \leq k$.

D.2 The Laws of Motion of the Endogenous State Variables

The laws of motion of the three endogenous state variables, which are inflation $\hat{\pi}_t$, real output \hat{y}_t , and the (nominal) interest rate \hat{R}_t , are given by the Euler equation (7), the Phillips curve

(6), and the Taylor rule (8). We want to write this system of linear equations as

$$\Gamma_0 \mathbf{s}_t = \Gamma_1 \mathbb{E}_t \mathbf{s}_{t+1} + \Gamma_2 X_{t|t}^{(0:k)}, \quad (36)$$

where $\mathbf{s}_t \equiv [\hat{\pi}_t, \hat{y}_t, \hat{R}_t]'$. It is obvious how to write equations (7) and (8) in the form (36). However, figuring out how to write the Phillips curve (6) in the form (36) requires a bit of work. First, note that given Propositions 1–2 and the equation $\widehat{mc}_{t|t}^{(k)} = \hat{y}_{t|t}^{(k)} - \hat{a}_{t|t}^{(k-1)}$, the imperfect-common-knowledge Phillips curve (6) can be rewritten as follows:

$$\begin{aligned} \mathbf{a}_0 X_{t|t}^{(0:k)} &= (1 - \theta) (1 - \beta\theta) \sum_{s=0}^{k-1} (1 - \theta)^s \mathbf{1}_2^T \left[\mathbf{v}_0 \mathbf{T}^{(s+1)} X_{t|t}^{(0:k)} \right] + \\ &- (1 - \theta) (1 - \beta\theta) \sum_{s=0}^{k-1} (1 - \theta)^s \left[\boldsymbol{\gamma}_a^{(s)'} X_{t|t}^{(0:k)} \right] \\ &+ \beta\theta \sum_{s=0}^{k-1} (1 - \theta)^s \mathbf{1}_1^T \left[\mathbf{v}_0 \mathbf{M} \mathbf{T}^{(s+1)} X_{t|t}^{(0:k)} \right], \end{aligned}$$

where $\mathbf{1}_1^T = [1, 0, 0]$, $\mathbf{1}_2^T = [0, 1, 0]$, and $\boldsymbol{\gamma}_a^{(s)} = [\mathbf{0}_{1 \times 5s}, (1, 0, 0), \mathbf{0}_{1 \times 5(k-s)}]'$. The following restrictions upon vectors of coefficients \mathbf{a}_0 and \mathbf{a}_1 can be derived from the rewritten Phillips curve:

$$\hat{\pi}_t = \left[(1 - \theta) (1 - \beta\theta) \left[\boldsymbol{\nu} \mathbf{m}_1 - \left(\sum_{s=0}^{k-1} (1 - \theta)^s \boldsymbol{\gamma}_a^{(s)'} \right) \right] + \beta\theta \boldsymbol{\nu} \mathbf{m}_2 \right] X_{t|t}^{(0:k)}, \quad (37)$$

where we define:

$$\begin{aligned} \mathbf{m}_1 &\equiv \begin{bmatrix} \mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(1)} \\ (1 - \theta) \left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(2)} \right] \\ \vdots \\ (1 - \theta)^k \left[\mathbf{1}_2^T \mathbf{v}_0 \mathbf{T}^{(k)} \right] \end{bmatrix}, \quad \mathbf{m}_2 \equiv \begin{bmatrix} \mathbf{1}_1^T \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(1)} \\ (1 - \theta) \left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(2)} \right] \\ \vdots \\ (1 - \theta)^k \left[\mathbf{1}_1^T \mathbf{v}_0 \mathbf{M} \mathbf{T}^{(k)} \right] \end{bmatrix}, \\ \boldsymbol{\nu} &\equiv \mathbf{1}_{1 \times k}. \end{aligned}$$

E Measuring Information Flows from the Signaling Channel

A salient feature of the dispersed information model is that the policy rate R_t transfers information about the output gap and inflation to price setters. We call the avenue by which this information is transferred the *signaling channel of monetary transmission*. Price setters use the

policy rate as a signal that helps them to track non-policy shocks (namely, technology shocks $\varepsilon_{a,t}$ and demand shocks $\varepsilon_{g,t}$) and, at the same time, to infer shocks to central bank's exogenous deviations from the monetary rule (i.e., monetary policy shocks $\varepsilon_{r,t}$). Following a standard practice in information theory (Cover and Thomas 1991), we use an entropy-based measure to assess how much information is provided by the signals firms observe in every period. The entropy measures the uncertainty about a random variable. For instance, the entropy associated with the level of aggregate technology \hat{a}_t , which is normally distributed with (unconditional) covariance matrix $var(\hat{a}_t)$, is defined as $H(\hat{a}_t) \equiv 0.5 \log_2 [2\pi e \cdot var(\hat{a}_t)]$.

We quantify the information flow conveyed by the signals as *the reduction of uncertainty* (i.e., entropy) at time t due to observing the signals in the information set $\mathcal{I}_{j,t}$.³⁷ For instance, the information flow about aggregate technology conveyed by the signals in the information set $\mathcal{I}_{j,t}$ can be computed as $\mathcal{H}(\hat{a}_t; \mathcal{I}_{j,t}) = H(\hat{a}_t) - H(\hat{a}_t | \mathcal{I}_{j,t})$, where the conditional entropy $H(\hat{a}_t | \mathcal{I}_{j,t}) \equiv 0.5 \log_2 [2\pi e \cdot var(\hat{a}_t | \mathcal{I}_{j,t})]$ and $var(\hat{a}_t | \mathcal{I}_{j,t})$ denotes the variance of aggregate technology conditional on firms having observed the signals in their information set $\mathcal{I}_{j,t}$.³⁸

We measure the information flow that firms receive about aggregate technology from observing *solely the private signals* as $\mathcal{H}(\hat{a}_t; \mathcal{I}_{j,t} / R^t) \equiv H(\hat{a}_t) - H(\hat{a}_t | \mathcal{I}_{j,t} / R^t)$ where $H(\hat{a}_t | \mathcal{I}_{j,t} / R^t)$ is the entropy conditional on firms having observed *only* their private signals. Endowed with this measure, we compute the fraction of private information about the aggregate technology \hat{a}_t as the ratio of the private information flow to the information flow from all the signals in the information set $\mathcal{I}_{j,t}$; that is, $\vartheta_a \equiv \mathcal{H}(\hat{a}_t; \mathcal{I}_{j,t} / R^t) / \mathcal{H}(\hat{a}_t; \mathcal{I}_{j,t})$. It should be noted that $\vartheta_a \in [0, 1]$. If ϑ_a is close to zero, then most of the information about aggregate technology stems from the policy signal. On the contrary, if ϑ_a is close to unity, then most of the information about aggregate technology stems from the private signal $\hat{a}_{j,t}$.³⁹ Analogously, we can define the fraction of private information about the demand conditions \hat{g}_t as $\vartheta_g \equiv \mathcal{H}(\hat{g}_t; \mathcal{I}_{j,t} / R^t) / \mathcal{H}(\hat{g}_t; \mathcal{I}_{j,t})$.

Let us define the entropy of aggregate technology conditional on firms having observed only the history of the policy signal as $H(\hat{a}_t | \hat{R}^t) \equiv 0.5 \log_2 [2\pi e \cdot var(\hat{a}_t | \hat{R}^t)]$, where $var(\hat{a}_t | \hat{R}^t)$ denotes the variance of aggregate technology conditional on firms having observed only the history of the policy signal R^t . We measure the information flow about aggregate technology conveyed only by the policy signal \hat{R}_t as $\mathcal{H}(\hat{a}_t; \hat{R}^t) \equiv H(\hat{a}_t) - H(\hat{a}_t | \hat{R}^t)$.

Another useful statistic for assessing the macroeconomic effects of the signaling channel

³⁷This approach is extensively followed by the literature of rational inattention pioneered by Sims (2003) and followed by Maćkowiak and Wiederholt (2009, 2015), Paciello and Wiederholt (2014), and Matejka (2016).

³⁸The units of the measure $\mathcal{H}(\hat{a}_t)$ are *bits* of information. The conditional variance can be pinned down by applying the Kalman-filter recursion, as shown in Appendix C. Note that having assumed that firms have received an infinitely long sequence of signals at any time t implies that the conditional covariance matrix $var(\hat{a}_t | \mathcal{I}_{j,t})$ is time invariant and is the same across firms at any time. Hence, information flows do not vary over time or across firms and we can omit indexing the information flow \mathcal{H} with j and t .

³⁹Note that the other private signal (i.e., $\hat{g}_{j,t}$) does not convey any information about the level of aggregate technology because of the assumed orthogonality of structural shocks at all leads and lags.

is the fraction of information about the exogenous state variables (i.e., \widehat{a}_t , \widehat{g}_t , $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) conveyed by the policy signal. For instance, the fraction of information about the level of aggregate technology \widehat{a}_t is computed as follows:

$$\Phi_a \equiv \frac{\mathcal{H}(\widehat{a}_t; \widehat{R}^t)}{\mathcal{H}(\widehat{a}_t; \widehat{R}^t) + \mathcal{H}(\widehat{g}_t; \widehat{R}^t) + \mathcal{H}(\widehat{\xi}_{m,t}; \widehat{R}^t) + \mathcal{H}(\widehat{\xi}_{\pi,t}; \widehat{R}^t) + \mathcal{H}(\widehat{\xi}_{x,t}; \widehat{R}^t)}, \quad (38)$$

where $\mathcal{H}(\widehat{a}_t; \widehat{R}^t) \equiv H(\widehat{a}_t) - H(\widehat{a}_t | \widehat{R}^t)$ measures the information flow about aggregate technology conveyed only by the policy signal \widehat{R}_t . $\mathcal{H}(\widehat{g}_t; \widehat{R}^t)$, $\mathcal{H}(\widehat{\xi}_{m,t}; \widehat{R}^t)$, $\mathcal{H}(\widehat{\xi}_{\pi,t}; \widehat{R}^t)$, and $\mathcal{H}(\widehat{\xi}_{x,t}; \widehat{R}^t)$ are the analogous objects for the demand conditions (\widehat{g}_t) and the components of the overall state of monetary policy ($\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$), respectively. The numerator quantifies the information flow about the level of aggregate technology \widehat{a}_t conveyed by the public signal. The denominator quantifies the information flow about the three exogenous state variables (i.e., \widehat{a}_t , \widehat{g}_t , $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) conveyed by the policy signal. This ratio Φ_a assumes values between zero and one. Analogously, we can define the fraction of information about the demand conditions conveyed by the policy signal as Φ_g and the fraction of information about the deviations from the monetary rule (i.e., $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, $\widehat{\xi}_{x,t}$) conveyed by the policy signal as Φ_m , Φ_π , and Φ_x . Note that $\Phi_a + \Phi_g + \Phi_m + \Phi_\pi + \Phi_x = 1$.

In summary, the ratio ϑ_a measures the accuracy of the private signal $\widehat{a}_{j,t}$ about the level of aggregate technology \widehat{a}_t relative to that of the policy signal. The ratios Φ_a , Φ_g , Φ_m , Φ_π , and Φ_x evaluate the accuracy of the public signal about each of the five exogenous state variables \widehat{a}_t , \widehat{g}_t , $\widehat{\xi}_{m,t}$, $\widehat{\xi}_{\pi,t}$, and $\widehat{\xi}_{x,t}$, respectively.

F Measurement Equations

The measurement equations are the following:

$$\ln \left(\frac{GDP_t}{POP_t^{\geq 16}} \right) - HPF \left[\ln \left(\frac{GDP_t}{POP_t^{\geq 16}} \right) \right] = \widehat{y}_t - \widehat{a}_t, \quad (39)$$

$$\ln \left(\frac{PGDP_t}{PGDP_{t-1}} \right) = \ln \pi_* + \widehat{\pi}_t, \quad (40)$$

$$FEDRATE_t = \ln R_* + \widehat{R}_t, \quad (41)$$

$$\ln \left(\frac{PGDP3_t}{PGDP2_t} \right) = \ln \pi_* + \widehat{\pi}_{t+1|t}^{(1)} + \varepsilon_t^{\mu_1}, \quad (42)$$

$$\ln \left(\frac{PGDP6_t}{PGDP5_t} \right) = \ln \pi_* + \widehat{\pi}_{t+4|t}^{(1)} + \varepsilon_t^{\mu_2}, \quad (43)$$

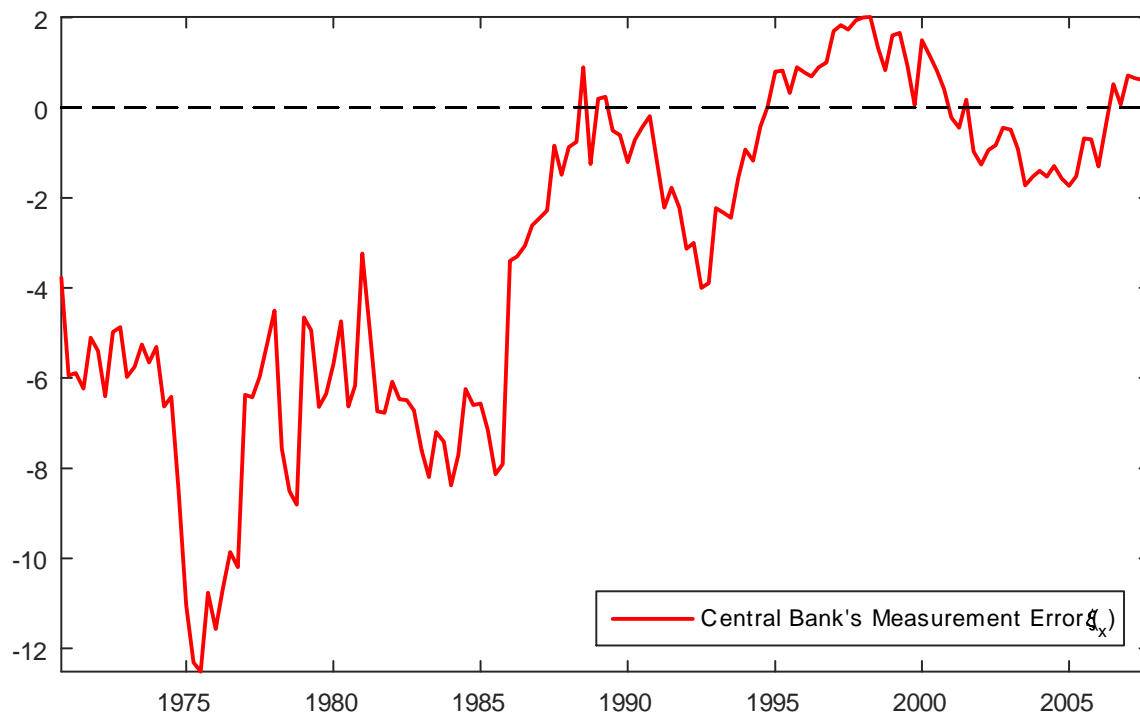


Figure 13: Federal reserve bank's measurement errors on nowcasting the output gap over the sample period.

$$\ln OGAP_t^{GB} = \hat{y}_t - \hat{a}_t + \hat{\xi}_{x,t}, \quad (44)$$

$$\ln INFL_t^{GB} = \ln \pi_* + \hat{\pi}_t + \hat{\xi}_{\pi,t}. \quad (45)$$

For these equations, $HPF \left[\ln \left(\frac{GDP_t}{POP_t^{\geq 16}} \right) \right]$ denotes the Hodrick–Prescott (HP) filter of real per-capita GDP, which is used to compute the potential output; GDP_t is the real gross domestic product computed by the U.S. Bureau of Economic Analysis (BEA) (Haver Analytics' mnemonic: *GDPC96*); $POP_t^{\geq 16}$ is the civilian non-institutional population aged 16 years old and over as computed by the U.S. Bureau of Labor Statistics (BLS) (Haver Analytics' mnemonic: *LNS10000000*); $PGDP_t$ is the GDP deflator computed by the BEA (Haver Analytics' mnemonic: *GDPDEF*); $FEDRATE$ is the average of daily figures of the effective federal funds rate (Haver Analytics' mnemonic: *FEDFUNDS*) reported by the Federal Reserve Economic Data (FRED) database managed by the Federal Reserve Bank of St. Louis; and $PGDP2_t$, $PGDP3_t$, $PGDP5_t$, and $PGDP6_t$ are the SPF's mnemonics for the median forecasts about the current, one-quarter-ahead, three-quarters-ahead, and four-quarters-ahead GDP price indexes, respectively. We relate these statistics with the first moment of the distribution of firms' expectations implied by the model.⁴⁰ To avoid stochastic singularity, we

⁴⁰A more coherent way to construct the data on inflation expectations is to compute the cross-sectional mean or median of the inflation forecasts of each individual forecaster. This measure would be closer to our model concept of the average first-order expectations. The Federal Reserve Bank of Philadelphia does release the individual forecasts with an ID to track each forecaster over time. However, I decided not to construct the series for the inflation expectations starting with these individual data because how the ID is assigned and managed,

introduce two Gaussian measurement errors $\varepsilon_t^{\mu_1} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\mu_1})$ and $\varepsilon_t^{\mu_2} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{\mu_2})$. The last two observables $\ln OGAP_t^{GB}$ and $\ln INFL_t^{GB}$ are the real-time data on the output gap and inflation from the Greenbook. These data are measured in *real time* (non revised) and capture the information set available to the Federal Open Market Committee. These series were constructed by Orphanides (2004) until 1995:Q4. I completed the data set using the tables kept by the Federal Reserve Bank of Philadelphia after harmonizing it.⁴¹ Figure 13 reports the dynamics of the real-time output gap from the Federal Reserve’s Greenbook data.

A quick look at equations (39) and (44) and at equations (40) and (45) shows that the two central bank’s measurement errors $\widehat{\xi}_{\pi,t}$ and $\widehat{\xi}_{x,t}$ are exactly pinned down by the data. In this respect, an appealing feature of having the HP filtered output gap among the observables is that the central bank’s measurement errors about the output gap exactly mimic the one in Orphanides (2004) when we estimate and evaluate the model. We believe that this helps obtain a neat assessment of the merits of the signaling channel in explaining the dynamics of inflation and inflation expectations relative to the merits of the mechanism proposed by Orphanides.

G Model Fit

In this section of the appendix, we compare the autocorrelation functions of the observable variables implied by the two competing models. Figure 14 plots the posterior mean of the autocorrelation functions implied by the two competing models for the seven observables against the sample autocorrelation function (red dashed line).⁴² These lines, which are often called *posterior predictive checks*, are obtained by simulating the model at each posterior draw for the model parameters, computing the statistic of interest, and averaging this statistic across posterior draws. The solid blue lines, which are the autocorrelation functions implied by the DIM, are always closer to the empirical autocorrelation functions (the red dashed lines) than the black circles, which stand for the autocorrelation functions implied by the PIM. In particular, one can observe that the PIM does a relatively poor job at accounting for the high persistence that characterizes the dynamics of inflation and inflation expectations in the sample. It should also be noted that the DIM captures remarkably well the high persistence of inflation expectations both at the one-quarter horizon and at four-quarter horizon. Both models cannot fully match the high empirical persistence of inflation, real-time inflation, and the federal funds rate.

especially before the Federal Reserve of Philadelphia took over, is unclear, casting serious doubts on whether these series are reliable. These concerns are detailed in Section 4 (Forecasts of Individual Participants) of the documentation of the SPF.

⁴¹The Federal Reserve Bank of Philadelphia computes the real time output gap as percent deviations of output Y_t from its potential Y_t^* (i.e., $100(Y_t - Y_t^*)/Y_t^*$). Therefore, these data must be adjusted so as to make them consistent with the data set constructed by Orphanides for the earlier quarters and with the model’s concept of output gap (i.e., $100(\ln Y_t - \ln Y_t^*)$). An analogous transformation is made for the real-time inflation rate.

⁴²The confidence bands are very tight for both models and hence are not reported.

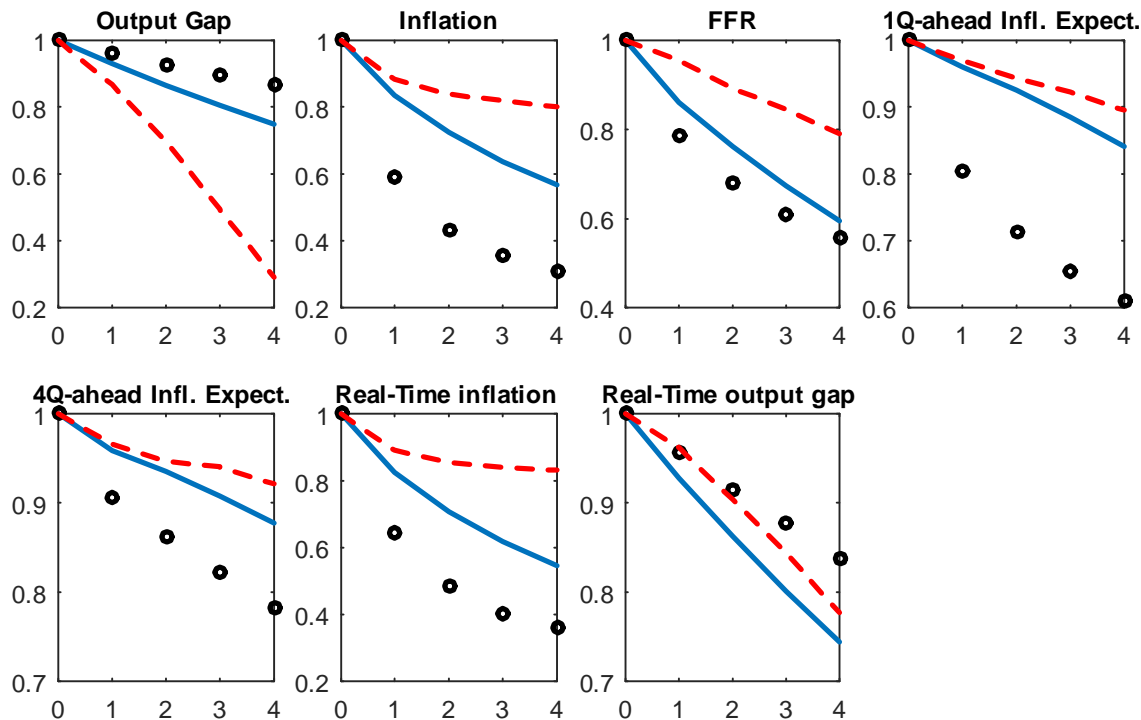


Figure 14: Autocorrelation functions for the seven observables. The red dashed line denotes the empirical autocorrelation function. The solid blue line denotes the autocorrelation function implied by the DIM. The black circles denotes the autocorrelation function implied by the PIM. The autocorrelation functions are computed by taking the mean of the autocorrelation functions evaluated at every 500 posterior draws.

However, the DIM seems to do a relatively better job at fitting the persistence of inflation and real-time inflation. Both models overpredict the persistence of the output gap. The two stylized models find it hard to reconcile the sizable difference in the persistence that characterizes the output gap and the real-time output gap. Furthermore, the DIM seems to do remarkably better than the PIM at fitting some cross-correlations, such as the cross-correlation between the two inflation expectations.

H Persistence of the VAR Response of Inflation and Expectations

Figure 15 shows the prior distribution for the half life of the response of inflation and inflation expectations to monetary shocks implied by the Bayesian VAR studied in Section 3.4.

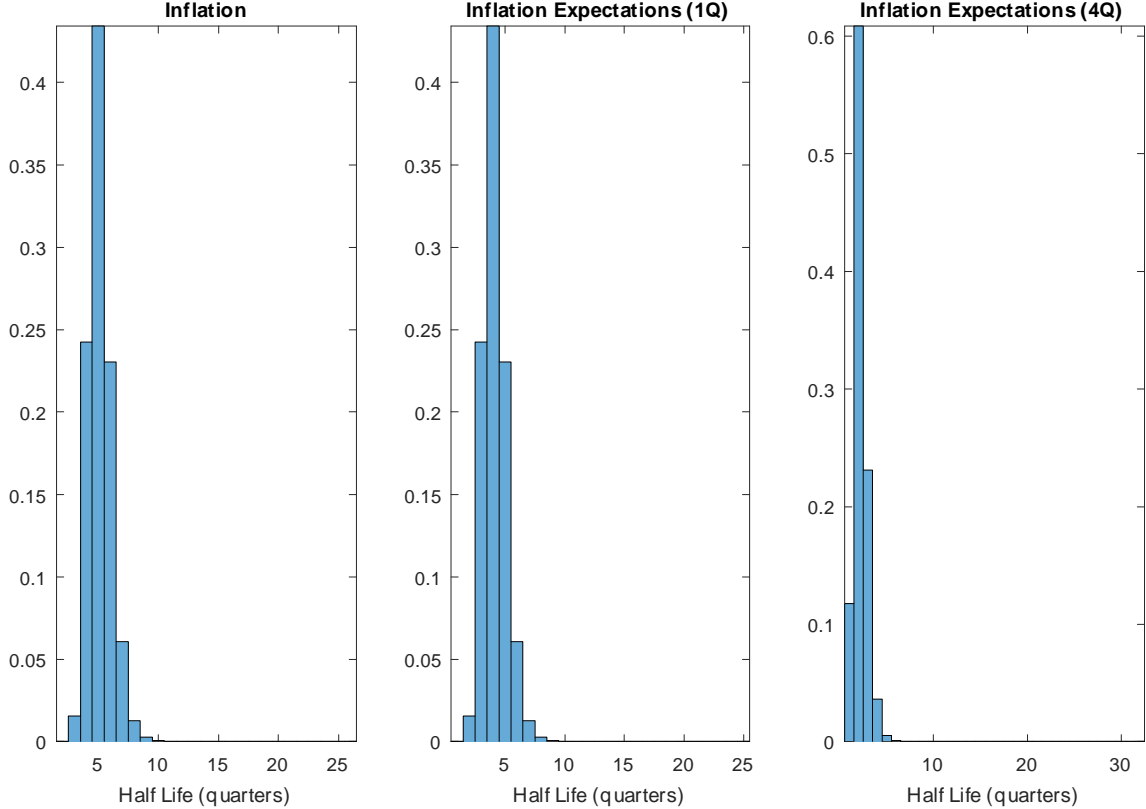


Figure 15: Prior distribution for the half lives of the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarter-ahead inflation expectations (right graph) to a monetary shocks in the model developed by Smets and Wouters (2007).

I Propagation of Technology Shocks

Figure 16 shows the response of the level of real output (GDP), inflation rate, the federal funds rate, one-quarter-ahead inflation expectations, and four-quarters-ahead inflation expectations to a one-standard-deviation positive aggregate technology shock. In the aftermath of a positive aggregate technology shock, real GDP increases, while both inflation and inflation expectations fall. The lower graphs of Figure 16 report the responses of average expectations about the five exogenous state variables. A drop in the policy rate owing to a positive technology shock induces firms to believe that the central bank is responding to either an expansionary deviations from the monetary policy rule ($\hat{\xi}_{m,t} < 0$ and $\hat{\xi}_{x,t} < 0$) or a negative demand shock. To the extent that firms are persuaded that an expansionary deviation from the rule has occurred, the negative response of inflation and inflation expectations to the technology shock will become smaller. This effect is non-negligible as the white bars lying in positive territory in Figure 17 show.⁴³ However, the monetary easing due to the positive technology shock leads firms

⁴³To improve the readability of the graphs, we have aggregated the contribution of the three exogenous deviations from the monetary rule (i.e., $\xi_{m,t}$, $\xi_{\pi,t}$, and $\xi_{x,t}$).

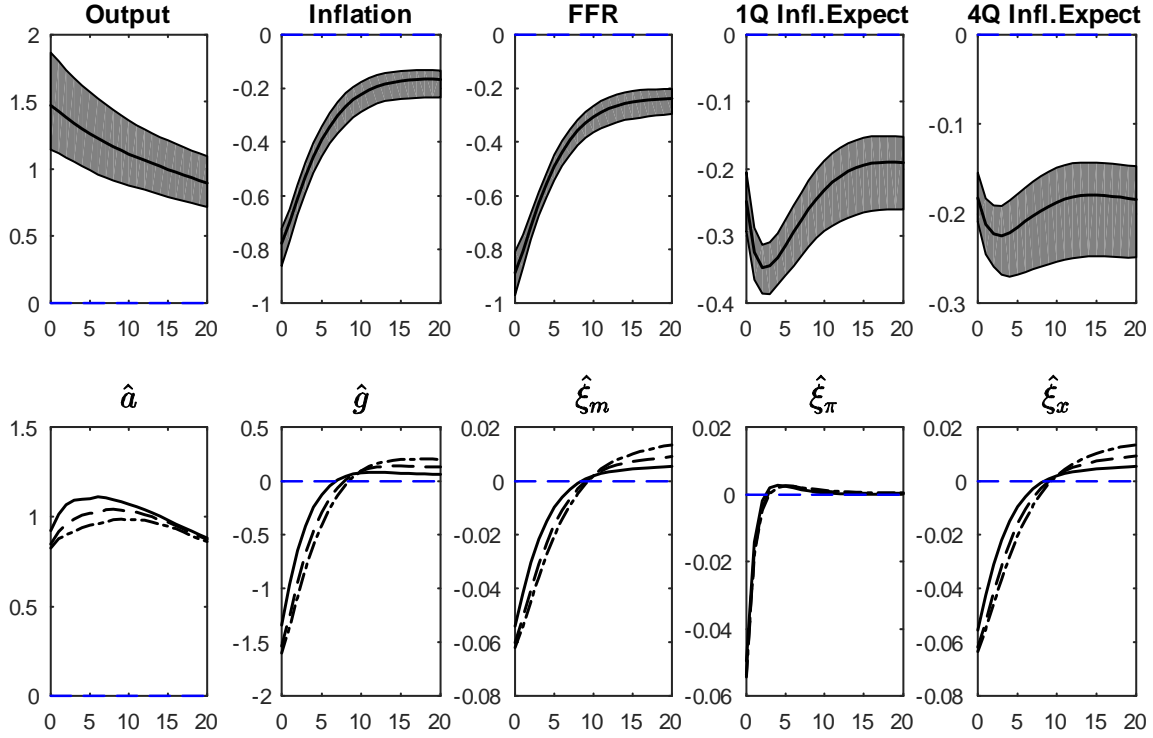


Figure 16: Impulse Response Functions to a Positive Technology Shock. *Upper graphs:* Impulse response functions of output, inflation, the federal funds rate (FFR), and one- and four-quarter-ahead inflation expectations in percentage deviations from their steady-state values to a one-standard deviation positive technology shock. The responses of inflation, the federal funds rate, and inflation expectations are annualized. The solid line denotes posterior means computed for every 200 posterior draws. The gray areas denote 90-percent credible sets. The horizontal axis in all graphs measures the number of quarters after the shock. *Lower graphs:* Responses of average expectations about the five exogenous state variables in percentage deviations from their steady-state level. Black solid lines denote the average first-order expectations. Dashed black lines denote the average second-order expectations. Dashed-dotted lines denote the average third-order expectations.

to believe that a negative demand shock may have hit the economy, lowering firms' inflation expectations and inflation. This effect is captured by the gray bars lying in negative territory in Figure 17.

It is interesting to notice that in Figure 17 the response of the average expectations about exogenous deviations from the monetary rule (i.e., the white bars) and those about the demand conditions (i.e., the gray bars) contribute to the adjustment of inflation and inflation expectations by similar amounts at any period after the technology shock. Two facts are behind this result. First, firms have inaccurate private information about demand shocks and, hence, have to mainly rely on the policy signal to learn about this type of shock. Second, the policy signal is mainly informative about technology shocks. The first condition implies that firms have to jointly learn demand shocks and exogenous deviations from the monetary rule by observing the policy signal. However, the second condition implies that the policy signal provides little

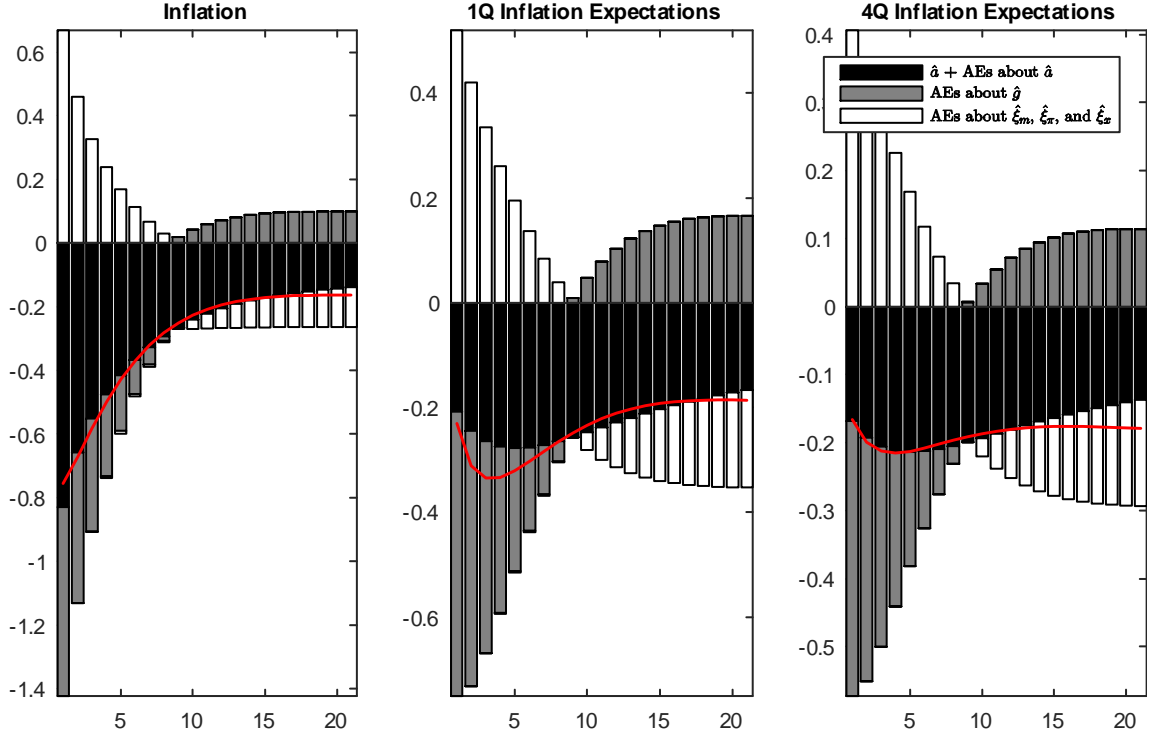


Figure 17: Contributions of average expectations to the impulse response functions of inflation and inflation expectations to positive technology shock. Parameter values are set equal to the posterior mean. The solid red line is the response of inflation (left graph), one-quarter-ahead inflation expectations (middle graph), and four-quarters-ahead inflation expectations (right graph). The vertical bars capture the contribution of the actual shocks and the average expectations about the level of aggregate technology \hat{a}_t , the demand conditions \hat{g}_t , and the three types of deviations from the monetary rule $\hat{\xi}_{m,t}$, $\hat{\xi}_{\pi,t}$, and $\hat{\xi}_{x,t}$ altogether to inflation and inflation expectations.

information about demand shocks and exogenous deviations from the monetary rule, making it quite hard for firms to figure out which of these shocks has prompted the central bank to raise the rate.

Figure 17 does not imply that signaling effects associated with technology shocks are necessarily tiny. In fact, the signaling channel also affects the contribution of the average expectations about the level of aggregate technology $a_t^{(0:k)}$ (i.e., the black vertical bars). Signaling effects associated with technology shocks will be precisely quantified in the next section. What Figure 17 shows is that the sluggish adjustment of inflation and inflation expectations in the aftermath of a technology shock is mainly due to the high persistence characterizing the average expectations about aggregate technology, which, as we shall see, the signaling channel significantly contributes to generate.

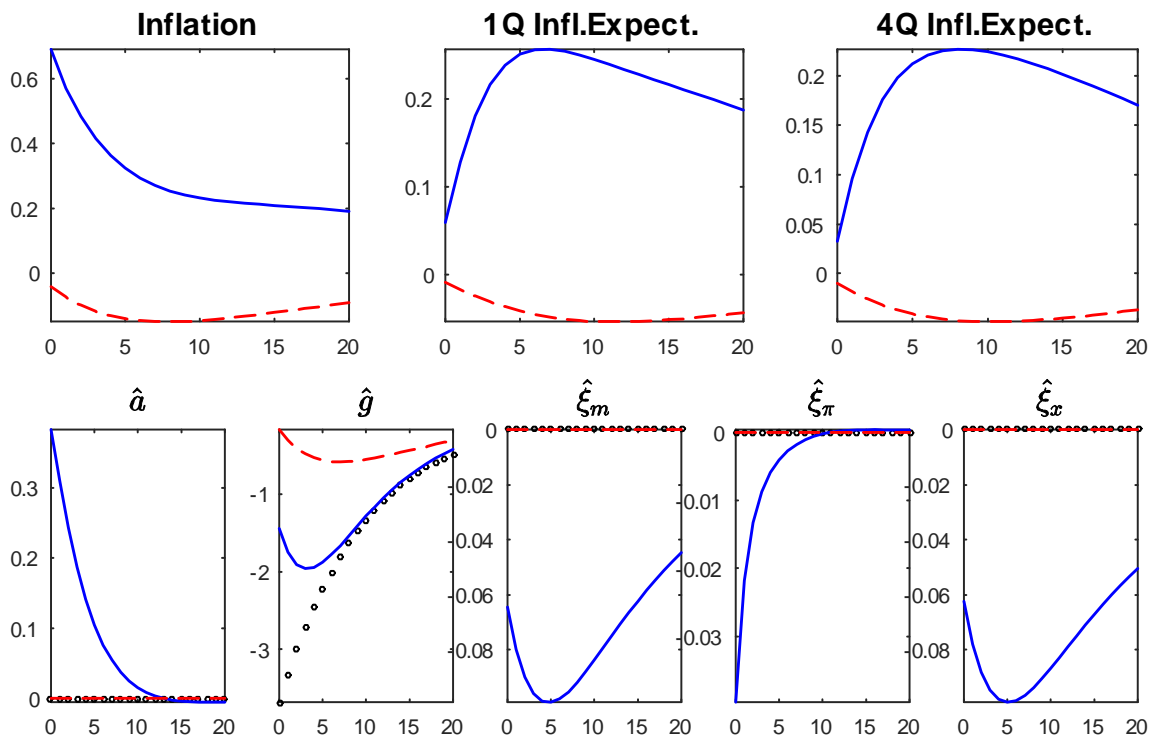


Figure 18: Signaling Effects on Inflation and Inflation Expectations Associated with a One-Standard-Deviation Negative Demand Shock. *Upper graphs:* Response of inflation (the left graph), one-quarter-ahead inflation expectations (the middle graph), and four-quarter-ahead inflation expectations (the right graph) in the estimated DIM, with signaling effects (the solid blue line) and in the counterfactual DIM, with no signaling effects (the red dashed line). *Lower graphs:* Black circles denote the response of the five exogenous state variables to a demand shock. The solid blue line denotes the response of the average first-order expectations about the five exogenous states in the estimated DIM, with signaling effects. The red dashed line denotes the response of the average first-order expectations about the five exogenous states in the counterfactual DIM, with no signaling effects.

J Demand-Driven Signaling Effects

The upper graphs of Figure 18 show that demand shocks give rise to persistent signaling effects on inflation and inflation expectations. The reason why these signaling effects on inflation and, in particular, on inflation expectations are so persistent is because negative demand shocks prompt the central bank to lower its policy rate by signaling expansionary deviations from the rule ($\hat{\xi}_{m,t}$ and $\hat{\xi}_{x,t}$). Those shocks are estimated to be fairly persistent. Firms are rational and thus expect that if the central bank deviates from the rule, this behavior will last for a fairly long time.

K Imperfectly Informed Households

To keep the model tractable enough to allow likelihood estimation, we assumed that only firms have limited information, whereas households are perfectly informed. Is assuming that

households have also limited information likely to overturn our finding regarding the relevance of signaling effects of monetary policy on inflation and inflation expectations in the 1970s? We argue that the answer is no. It should be noted that the dispersed information model shares with the stylized perfect information New Keynesian DSGE models a recursive structure: the Phillips curve determines inflation given firms' expected path of the output gap, whereas the Euler equation determines the output gap given a path of the natural rate⁴⁴ $\hat{r}_t^n = (1 - \rho_g) \hat{g}_t - (1 - \rho_a) \hat{a}_t$ and the actual real rate $\hat{R}_t - E_t \hat{\pi}_{t+1}$. Note that the natural rate solely depends on exogenous non-policy disturbances. Expanding the Euler equation forward in the DIM leads to $\hat{x}_t = - \sum_{k=0}^{\infty} \left(\hat{R}_{t+k} - E_t \hat{\pi}_{t+k} - \hat{r}_{t+k}^n \right)$. Assuming that households have incomplete information changes the Euler equation to the following one: $x_t = - \sum_{k=0}^{\infty} \left(\hat{R}_{t+k|t}^{(1)} - \hat{\pi}_{t+k|t}^{(1)} - \hat{r}_{t+k|t}^{n(1)} \right)$.⁴⁵ In our discussion, we assume that households have the same information set as firms; that is, $\mathcal{I}_{j,t}$ for $j \in (0, 1)$ in (5) and, hence, the average expectations about the nominal rate are equal to the true rate; that is, $R_{t|t}^{(1)} = R_t$.

As shown in Section 3.6, signaling effects associated with demand shocks are key in explaining why inflation and inflation expectations were persistently heightened in the 1970s. Relaxing the assumption of perfectly informed households gives rise to three effects on the response of inflation in the aftermath of a negative demand shock. First, abstracting from the information received through the policy signal \hat{R}_t , households have imperfect *private* information about the demand conditions \hat{g}_t , which affects the dynamics of the natural rate of interest. Imperfect private information implies that the expected path of the future natural rates would fall less after a negative demand shock as households' beliefs will respond less than the actual variables to the shock, as shown in Figure 18.⁴⁶ Second, signaling effects cause households' expectations about future demand conditions $\hat{g}_{t+h|t}^{(1)}$ to fall more and expectations about the dynamics of aggregate technology $\hat{a}_{t+h|t}^{(1)}$ to rise. This can be seen by comparing the solid blue line and the red dashed line in the first two lower graphs from the left in Figure 18. Hence, signaling effects lower the expected path of the future natural rates. Therefore, these two effects on the expected future path of the natural rate go in opposite directions, suggesting that the assumption of perfectly informed households does not necessarily overstate the magnitude of signaling effects on inflation and inflation expectations in the 1970s. Third, the signaling channel leads households to expect future expansionary deviations of the policy rate from the monetary rule. See the

⁴⁴The natural interest rate is the real interest rate that would arise in the model under perfect information and no nominal rigidities.

⁴⁵We abstract from technical complications that would make this extension of the model impossible to evaluate using only pencil and paper, such as the fact that having heterogenous households would lead the distribution of bond holdings to enter the state vector of the economy. Furthermore, we follow standard assumptions in the literature that studies models with learning to ensure that the aggregate resource constraint is satisfied.

⁴⁶Compare the red dashed line that captures the response of the average first-order expectations about the demand conditions when firms observe only private signals (i.e., the signaling channel is shut down) to the black circles that capture the response of the actual demand conditions in the second-from-left lower graph.

lower graphs in Figure 18. These beliefs would imply an even lower expected path for the policy rate. The Euler equation would then imply that the output gap would be reduced and, hence, inflation and inflation expectations are expected to rise even more than they would in the case of perfect information. Thus, assuming that households have also imperfect information does not necessarily lower the signaling effects on inflation and inflation expectations associated with demand shocks and could even magnify these effects.